Large Hierarchies in String Compactifications

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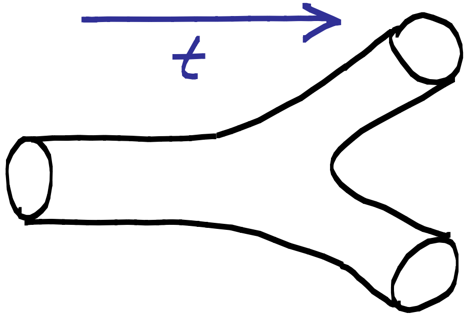
Outline

- Type IIB supergravity
- Fluxes and the cosmological constant problem
- The landscape "solution"
- "Throats" and the Randall-Sundrum model
- Statistical predictions for throats & large hierarchies

String Theory vs. Supergravity



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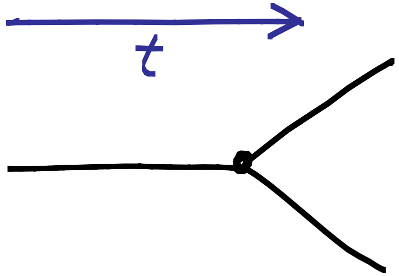


(particle decay)

5 consistent quantization prescriptions in $d=10$ ⟷ 5 consistent 10d low-energy effective field theories (supergravities)

(Type I, Type IIA, Type IIB,
heterotic $SO(32)$, heterotic $E_8 \times E_8$)

Type IIB supergravity lagrangian

Fields:

- 10d metric $g_{\mu\nu}$
- dilaton φ
- form fields C_0, C_2, B_2, C_4

  (C_p is an antisymm. tensor of rank p, e.g.,

  $C_0 =$ scalar, $C_2 = (C_2)_{\mu\nu}\, dx^\mu \wedge dx^\nu$, etc.)

- fermionic partners

$$S = \int d^{10}x \sqrt{-g} \left\{ e^{-2\varphi} \left(R + (\partial\varphi)^2 \right) + F_1^{\,2} + |G_3|^2 + F_5^{\,2} + \cdots \right\}$$

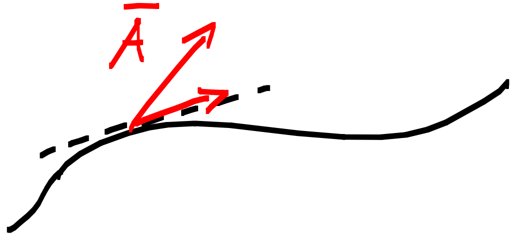
where, e.g., $(F_5)_{\mu_1 \cdots \mu_5} = \partial_{\mu_1} (C_4)_{\mu_2 \cdots \mu_4}$ + cycl. perm.

(G_3 combines $F_3 = dC_2$ & $H_3 = dB_2$)

Differential forms (a reminder)

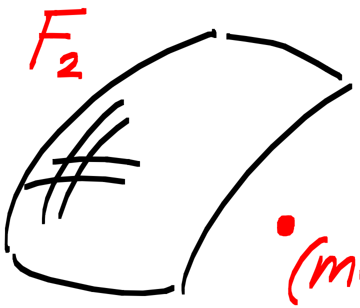
- A 1-form $(A_1)_\mu$ can be integrated along a line:

$$\int A_\mu \, dx^\mu \; :$$



- A 2-form $(F_2)_{\mu\nu}$ can be integrated along a surface:

$$\int (F_2)_{\mu\nu} \, dx^\mu \wedge dx^\nu \; :$$



← part of surface integral

• (magnetic) charge

The antisymmetry makes

$$\int (F_2)_{\mu\nu} \, dx^\mu \wedge dx^\nu \equiv \int (F_2)_{\mu\nu} \frac{\partial x^\mu}{\partial \sigma^1} \frac{\partial x^\nu}{\partial \sigma^2} \, d\sigma^1 d\sigma^2$$

independent of the parametrization of the surface by σ^1, σ^2.

Compactification to 4 dimensions

- 6 of the 10 dimensions of type IIB supergravity have to be "curled up" with small radius

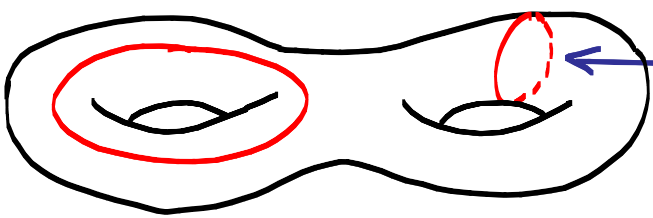
- simplest examples: Calabi-Yau spaces

  crucial feature for us: Ricci-flat metric, i.e.

  $$R_{\mu\nu} = 0 \quad (\text{while } R_{\mu\nu\delta}{}^{\sigma} \neq 0 \text{ in general})$$

  $\Rightarrow$ Einstein-equ. solved without matter

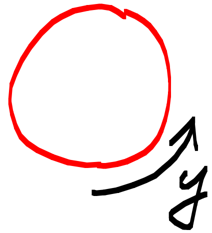
- Calabi-Yaus can have complicated topology (many cycles)



example of "non-trivial 1-cycle" (1-dim. submanifold)

Simplest example of a "flux Background"

- Consider S^1 as a 1-dim. compact space:

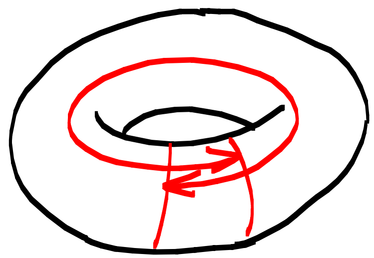


- Consider 0-form $\varphi(x^0, \cdots, x^3, y)$ with a 1-form - field strength $\partial_\mu \varphi(x^0, \cdots, x^3, y)$
- The S^1 geometry normally implies $\varphi(y) = \varphi(y + 2\pi)$
- Let us instead demand $\boxed{\varphi(y) = \varphi(y + 2\pi) + C}$

  - $\varphi(y) = y \cdot C / (2\pi)$ is a solution
  - The field strength $\partial_y \varphi = C/(2\pi)$ implies a non-zero energy density $(\partial_\mu \varphi)^2 = \frac{C^2}{4\pi^2}$
  - No smooth transition to $\varphi \equiv 0$ possible

Another simple example : 2-form flux on a torus

- parameterize torus by x^5, x^6 $\left(x^5 \sim x^5 + 1 \,;\, x^6 \sim x^6 + 1 \right)$
- let $A_5 = 0$ and $A_6 = c \cdot x^5$



gauge trf. needed

- $F_{56} = \partial_5 A_6 - \partial_6 A_5 = c$

  (solves equ. of motion)

- the existence of a global wave-fct. for the electron requires flux quantization

$$\int_{T^2} F_2 = 2\pi N$$

- More generally :

$$F_p = d\, C_{p-1} \text{ locally} \; ; \quad \boxed{\int_{p\text{-cycle}} F_p = \text{const.} \times N}$$

The cosmological constant problem

- Calabi-Yau compactification $\Rightarrow$ supersymmetric 4d-world, many massless scalars (parameterizing shape of CY)

$$\boxed{\Lambda_4 = 0}$$

- SUSY-breaking (introduced by non-SUSY fluxes; non-perturb. effects; presence of anti-branes etc.)

$$\boxed{\Lambda_4 \sim M_{\text{string}}^4}$$

(This is the generic expectation; a concrete model might have $\Lambda_4 = 0$ after SUSY is broken, <u>but no good reason for such an "accident" is known</u>)

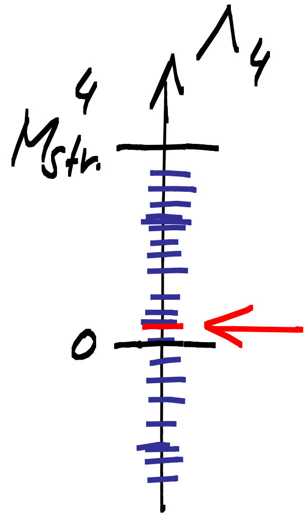
The landscape "solution" to the cosm. const. problem

- Typical Calabi-Yaus have ~ 100 3-cycles
- Assume that each cycle supports F_3-flux $\int F_3 \sim N$ with $N \lesssim 10$ (and similarly for H_3)

$\Rightarrow \quad 10^{2 \cdot 100} \sim \underline{10^{200} \text{ distinct "vacua"}}$

- The 4d energy densities vary because of

$$S \supset -\int (F_3)_{\mu\nu\rho} (F_3)^{\mu\nu\rho}$$

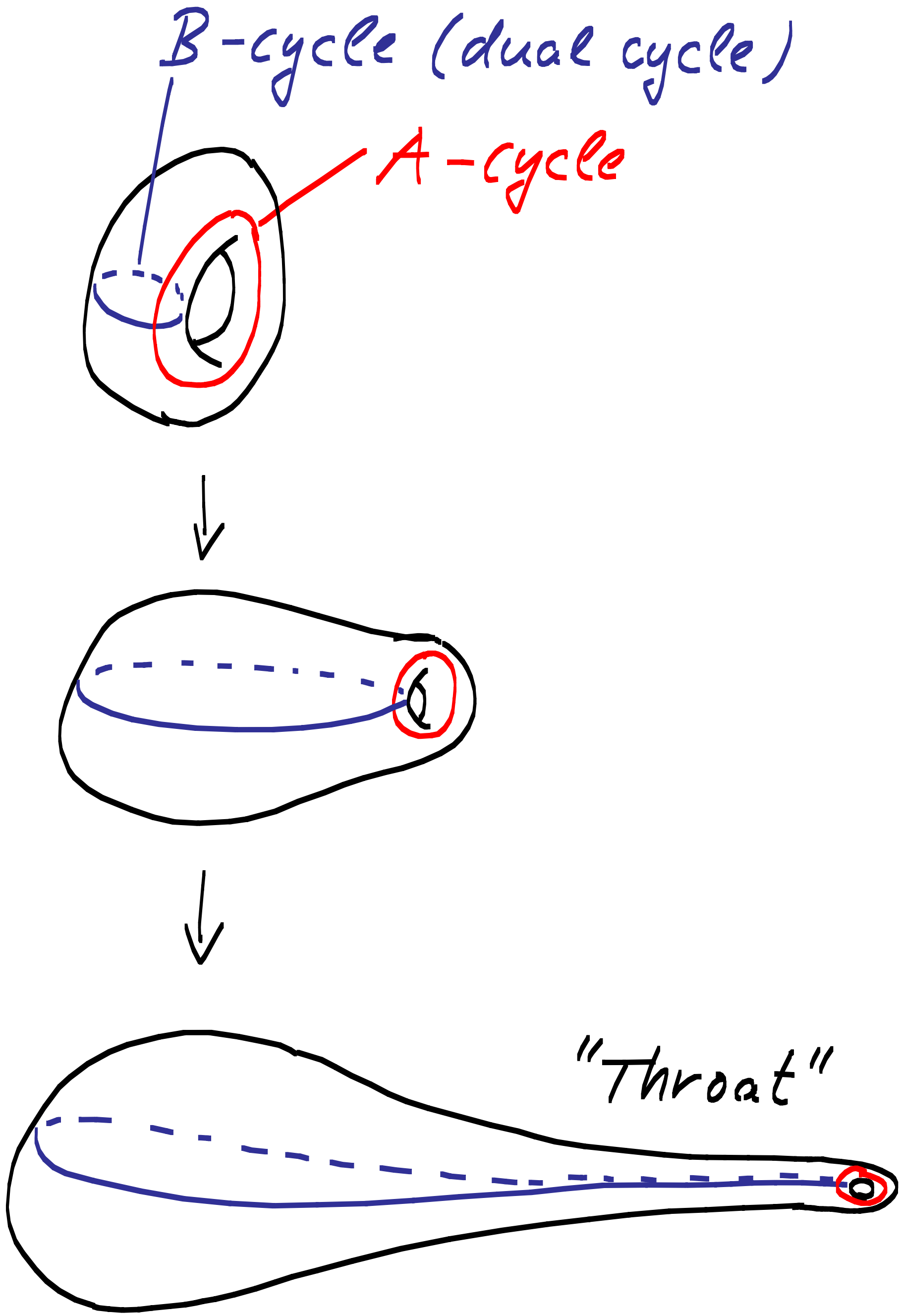


- If the energy levels are randomly spread, we expect

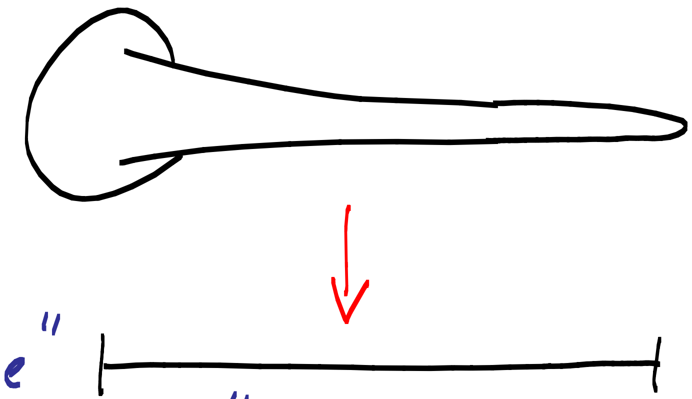
$$\boxed{\Lambda_{4,min} \sim 10^{-200} M_{string}^4}$$

"Throats"

- Consider a space with two 3-cycles:
  (we can only draw 1-cycles)
- The fluxes stabilize the cycle-volume
  (more flux → larger volume)
- A large ratio of fluxes on B & A-cycle can lead to
  "throat geometries"



This "throat geometry" allows for a 5d interpretation:

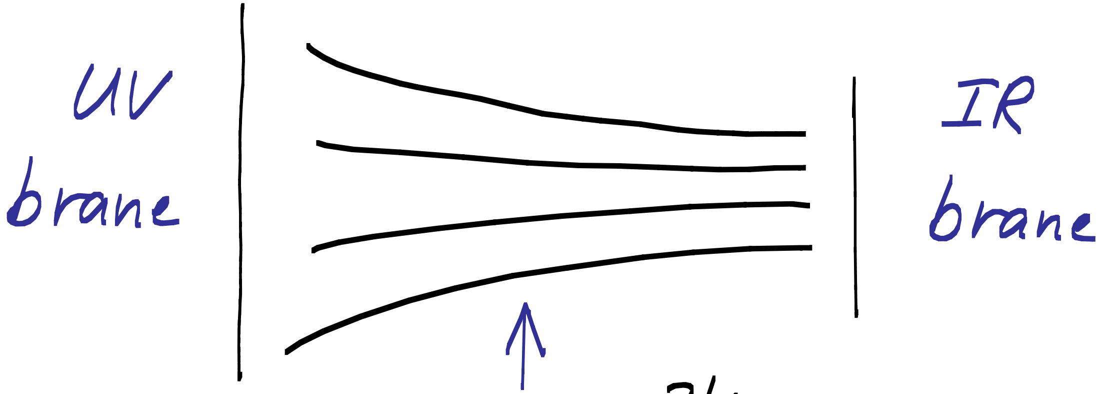


- Geometry of throat region: $\sim AdS_5 \times T^{1,1}$ $\quad (T^{1,1} \sim S^2 \times S^3)$
- Effective 5d geometry:

$$ds^2 = \underbrace{e^{2A(y)}}_{\text{"warp factor"}} dx_\mu dx^\mu + dy^2$$

This is very similar to the famous

"Randall-Sundrum model"



$$ds^2 = e^{2ky}dx^2 + dy^2$$

with Lagrangian

$$\mathcal{L} = \frac{1}{2} M_5^3 \mathcal{R} - \Lambda_5 + \delta(y - y_{UV/IR})\,\mathcal{L}_{UV/IR}\left(\psi_{UV/IR},\, g_{4,UV/IR}\right)$$

induced from 5d bulk

$\Rightarrow$ exponential hierarchy of scales $\sim \underline{g_{4,UV}/g_{4,IR}}$

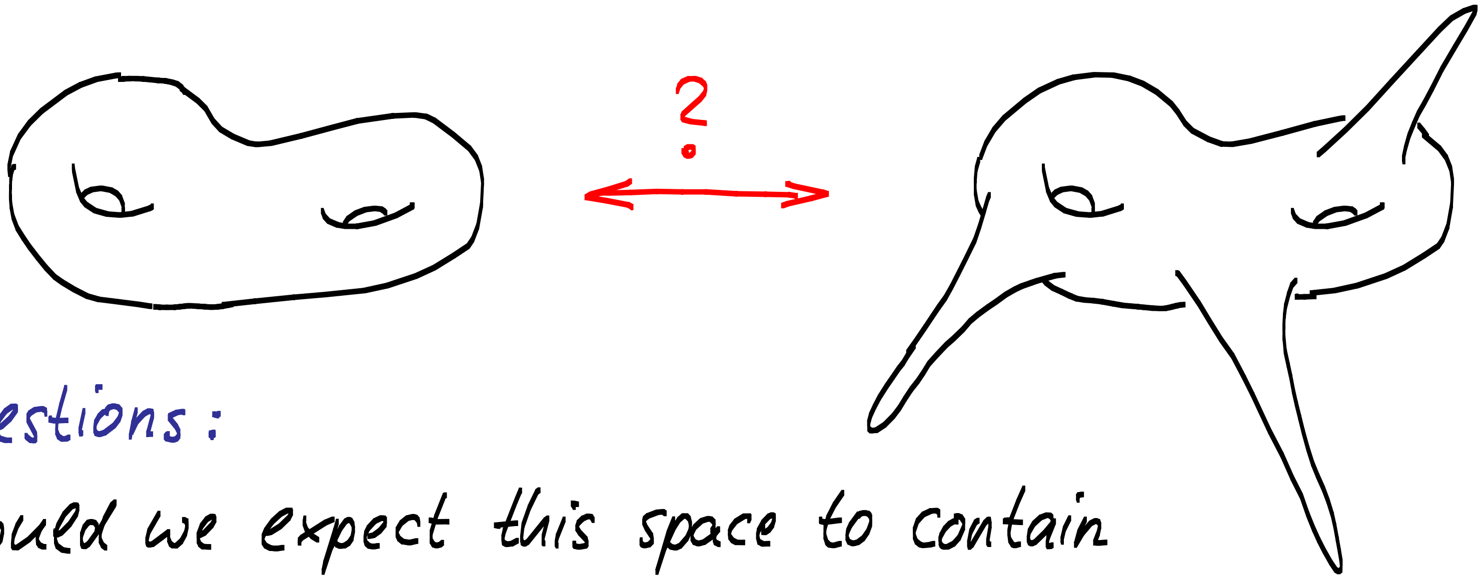
In summary:

- Type IIB models on Calabi-Yau (-orientifolds) with many cycles & fluxes may "solve" the cosmological constant problem
- In this context "Klebanov-Strassler throats" appear naturally and may induce large hierarchies à la Randall-Sundrum

$$\boxed{m_{IR} \sim e^{2A(y)_{min}} M_{string} \sim \begin{pmatrix} \text{warp} \\ \text{factor} \end{pmatrix} \times M_{string}}$$

- For details: Bousso / Polchinski '00
  Giddings / Kachru / Polchinski '01 ("GKP")
  Kachru / Kallosh / Linde / Trivedi '03 ("KKLT")

Let us now quantify the statement that "throats are common in the type IIB landscape".
(→ recent paper "The Ubiquitous Throat" with J. March-Russell)



Questions:

- Should we expect this space to contain strongly warped regions (Klebanov-Strassler-throats)?
- How many of them?
- Which warp factors?

⇒ Important consequences for cosmology, SUSY-Breaking etc. may follow ...

Basic idea of analysis

- Expect orientifold with many 3-cycles (since otherwise the choice of fluxes will be too limited to allow for a sufficiently small cosm. constant Λ)
- Random flux numbers $\Rightarrow$ Some 3-cycles carry small flux numbers $\Rightarrow$ those cycles stabilized at small volume
- If the zero-volume limit of a cycle gives conifold point, then the small-volume case gives "throat"

$\Rightarrow$ Distribution of number & length of throats becomes a well-defined statistical question (→ Douglas et al.) (assuming no correlation with the fine-tuning of Λ)

- Result: Binomial distribution

The number of 3-cycles

- consider CY-orientifold with K 3-cycles
- flux vector: $N \in \mathbb{Z}^{2K}$
- "Gauß' law":

[Euler number of F-theory 4-fold]

$$\underbrace{\frac{\chi_4}{24}}_{L_*} = \underbrace{\frac{1}{2} N^T \Sigma N}_{L} + N_{D3} \,, \quad \Sigma = \begin{pmatrix} 0 & 1\!\!1 \\ 1\!\!1 & 0 \end{pmatrix}$$

($N_{D3} > 0$ to preserve SUSY)

- number of vacua: $\boxed{\mathcal{N}_{SUSY}(L \le L_*) \sim L_*^K / K!}$ (→ Denef, Douglas)

- more specifically: We want realistic vacua with small cosmol. constant and ~~SUSY~~.

$$\left(\text{no-scale} \xrightarrow[\text{non-pert.}]{} \text{SUSY-AdS} \xrightarrow[\overline{D3}\text{-uplift}]{} \cancel{\text{SUSY}} \right)$$

$$\mathcal{N}_{uplift} = \mathcal{N}_{SUSY}\left(L_* < L < L_* + N_{\overline{D3},\,max}\right)$$

$$\sim \underbrace{N_{\overline{D3},\,max}}_{\sim 0.08 \cdot \left(\begin{matrix}\text{RR-flux on}\\ \text{throat cycle}\end{matrix}\right) \ll L_*} \frac{L_*^{K-1}}{(K-1)!} \sim \frac{L_*^{K}}{K!}$$

(Result is parametrically the same as for SUSY vacua.)

- Λ_4 in uplifted vacua varies from $-|W|^2$ term in scalar potential
- $W = \int G_3 \wedge \Omega \implies$ uniform distribution in complex W plane $\implies$ flat distribution of $-|W|^2$

In detail:

- # of vacua with $|W_0| < |W| < |W_0| + |\Delta W|$ is $\sim$ area of a ring : $\sim |W_0| \cdot |\Delta W|$
- This is $\sim \Delta |W|^2$, i.e., same # of vacua in every $\Delta |W|^2$-intervall

- Since, typically, the $-|W|^2$ contribution is $\sim M_{string}$, the chance to hit the right Λ_4-region is

$$\frac{\Lambda_{4,obs.}}{M_{string}^4} \sim 10^{-120} \implies \text{need } \mathcal{N} \sim 10^{120}$$

$$\implies \boxed{K \sim \log(10^{120}) / \log(eL_* / \log(10^{120}))}$$

⇒ conservative estimate: choose L_* as large as possible ($L_* \sim 10^4$)

⇒ $\boxed{K \sim 60}$

alternative (less conservative) estimate:

- CYs with 200 ⋯ 400 3-cycles are "typical"
- large number of cycles statistically preferred

⇒ $\boxed{K \sim 400/2 \sim 200}$

<u>Crucial assumption</u>: Many of these 60 ⋯ 200 3-cycles can shrink to <u>conifold singularity</u>

(3-cycles producing this singularity are appropriate for throats ; the singularities arise at points in the "<u>moduli space</u>" of CYs – parameterized by <u>$z_i \in \mathbb{C}$</u>)

Stability issues

moduli: $\phi_a = (\tau, z_i)$ $\quad i = 1 \cdots K/2$

$$V = e^K \Big(\underbrace{K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W}}_{\text{positive mass matrix}} - \underbrace{3|W|^2}_{\text{potentially negative terms}} \Big)$$

SUSY vacua:

$D_a W = 0$

(vanish for $W_0 \to 0$)

$\Rightarrow$ expect no stability problems after uplift

However: Denef/Douglas find stability problems in near conifold case for 1 compl. structure modulus

What is the reason? Are these problems generic?

Small W_0, Small $\delta\phi^a$

$$\Rightarrow \quad V \sim \delta\phi^a \underbrace{W_{ab}} K^{b\bar{c}} \underbrace{\bar{W}_{\bar{c}\bar{d}}} \delta\phi^{\bar{d}}$$

2nd derivative matrices

Recall: $W = A(z) + \tau B(z)$; $\int \Omega \sim z \ln z$

near conifold point

$$\Rightarrow \quad W_{ab} \sim \begin{pmatrix} 0 & \sim 1 \\ \sim 1 & 1/z \end{pmatrix}$$

$\Rightarrow$ 1 small eigenvalue; stability problem!

However: 2 compl. structure moduli

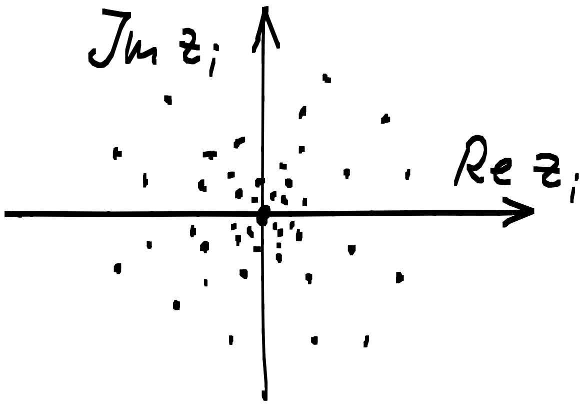
$$\Rightarrow \quad W_{ab} \sim \begin{pmatrix} 0 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & \sim 1 \\ \sim 1 & \sim 1 & 1/z \end{pmatrix}$$

$\Rightarrow$ no small eigenvalue;

<u>no stability problems!</u>

Distribution of throats

Denef/Douglas: probability for being near conifold point $z_i = 0$



$$p_i(|z_i|) \simeq \frac{1}{c_i \ln(1/|z_i|)}$$

$O(1)$ constant depending on moduli space away from $z_i = 0$

Giddings/Kachru/Polchinski:

warp factor (throat hierarchy) $\boxed{h_i \sim |z|^{-1/3}} \sim e^{N_B/N_A}$

The various conifold points represent $\sim K$ subspaces of co-dimension one in the complex $K/2$-dimensional moduli space. Throats are in slices around them.

- Probability for creating hierarchy $> h_i$ at the $z_i \to 0$ conifold point is

$$p_i(h_i) \simeq \frac{1}{3 c_i \log h_i}$$

- Probability for creating n throats with hierarchy $h > h_*$ is

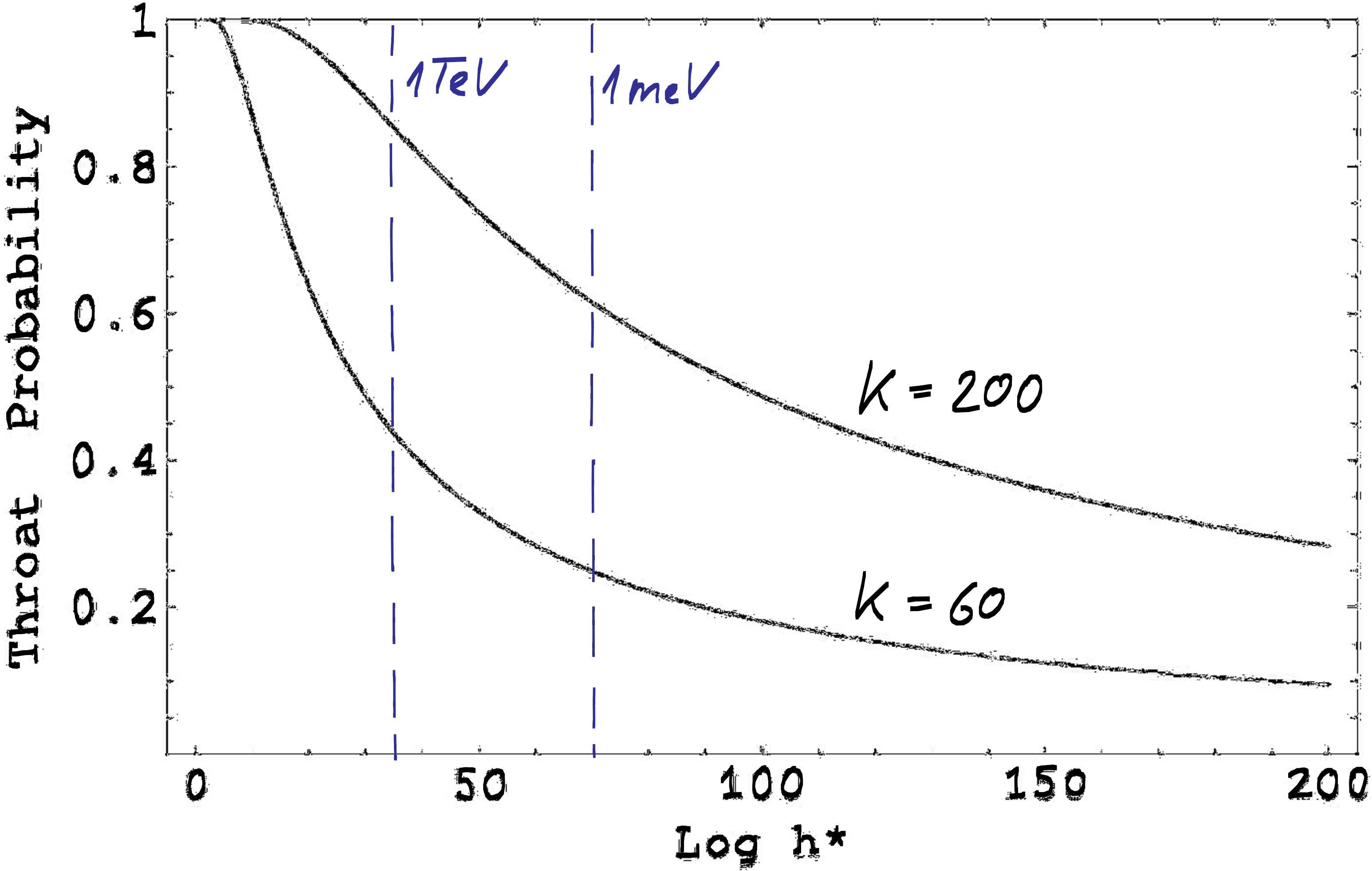
$$\boxed{p(n, h>h_* | K) \sim \binom{K}{n} p^n (1-p)^{K-n}} \; ; \quad p \equiv \frac{1}{3c \log h_*}$$

(Binomial distribution)

$\Rightarrow$

- Throat # with $h > h_*$: $\quad \bar{n}(h>h_* | K) = \dfrac{K}{3c \log h_*}$
- Longest expected throat: $\quad h_1 \sim \exp(K/3c)$
- Probability for no throat with hierarchy $h > h_*$: $\quad p(0, h>h_* | K) \sim \exp\left(-\dfrac{K}{3c \log h_*}\right)$

Another interesting quantity: Probability $P(h > h_* | K)$ for having at least one throat with hierarchy $> h_*$



Both with $c = 1$

$\Longrightarrow$ Very low "price" for having even a very long throat

Towards phenomenology

or: Are throats really common?

Main problem: The parameters K and c are not known.

① Conservative scenario:

$K = 60$ (minimal value for cosm. constant) ; $c = 3$

$\Rightarrow$ crucial combination: $K/3c \sim 7$

$\rightarrow$ largest expected hierarchy: 10^3

$\rightarrow$ expect ~ 3 throats with hierarchy 10 or larger

(may be interesting for inflation etc., but no striking low-energy phenomenology)

$\rightarrow$ however: $\bar{n}\,(h > 10^{15}) \sim 0.2$ $\Rightarrow$ "electroweak hierarchy throat" in 1 out of 5 vacua.

② Favourable scenario: $K = 200$; $c = 1/3$

$\Rightarrow K/3c = 200$

→ largest expected hierarchy: 10^{80}

→ not having a throat with $h > 10^{30}$ (meV-scale!) has only 5% probability (such throats are a prediction!)

→ expect ~6 electroweak hierarchy throats

More specifically:

KKLT setup; $\overline{D3}$ branes in the throat are the only source of SUSY-breaking; Low-scale SUSY needs a warp factor $\sim 10^7$

The required "10^7-throat" is present in 50% of vacua!

Conclusions / Outlook

- We have attempted to quantify the statement hat

  "throats are common in the type IIB landscape".

- Details of outcome depend on number of 3-cycles (K) and on the geometry of moduli space (c)

- $K/3c$ → conservative: expect $h \sim 10^3$; $h \sim 10^{15}$ in 20% of vacua

  → favourable: expect $h \sim 10^{80}$;

  "meV-throats" are a firm prediction

- Need better understanding of geometry of CY 3-cycles and of moduli space

- If "favourable case" confirmed, could throats rule out the type IIB landscape via cosmological bounds?