String Phenomenology

(A. Hebecker, Heidelberg)

(idealized) Plan:

- String Theory, Superstring Theory
- 10 d supergravity, Example: type IIB
- Diff. forms, homology/cohomology
- Compactification to 4d, Kaluza-Klein theory
- CY moduli space (torus as an example), 4d effective theory
- Fluxes, GKP/KKLT, "LARGE-volume", the landsape
- D7-branes, Dualities/M-theory, F-theory

(Alternatives: D3 on singularities, IIA, heterotic, G_2, Gepner models)

Some Books: Green / Schwarz / Witten ; Polchinski ; Becker / Becker / Schwarz ; Kiritsis ; Zwiebach ; (Blumenhagen) / Lüst / Theisen

Some reviews: Quevedo – hep-th/9603074

Douglas / Kachru – hep-th/0610102 ; Denef – 0803.1194

See also: www.personal.uni-jena.de/~p5thue2/notes/strings.html

Why string theory?

- QFT: UV-divergences, e.g.

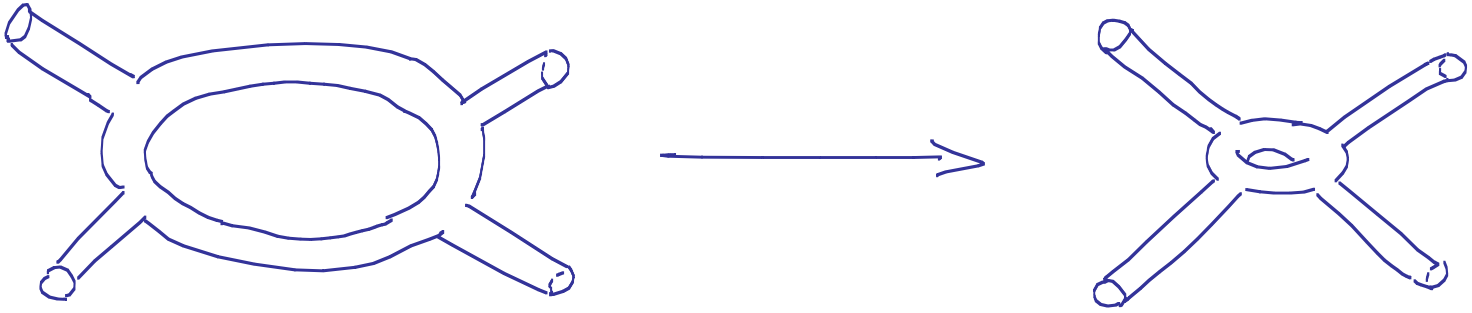
- Problem comes from

$G(x-y) \sim \frac{1}{(x-y)^2}$, i.e. from "point particle interactions".

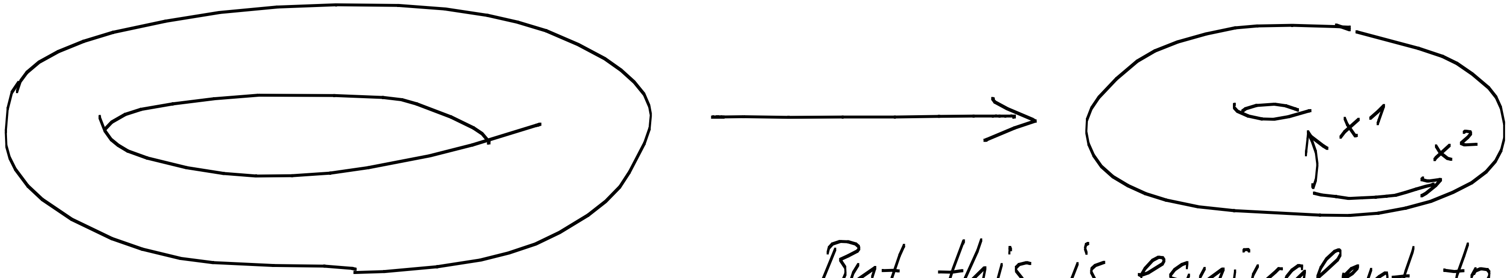
- The usual renorm. procedure ($\Lambda \to \infty$ at fixed σ's) does not work in gravity

[Cf. however LQG; UV-safety; Lattice etc. Unfortunately, so far all these proposals have technical problems and no implications for particle physics are in sight.]

- String Theory: No UV divergences (including, in particular, gravity)



- simplified loop:

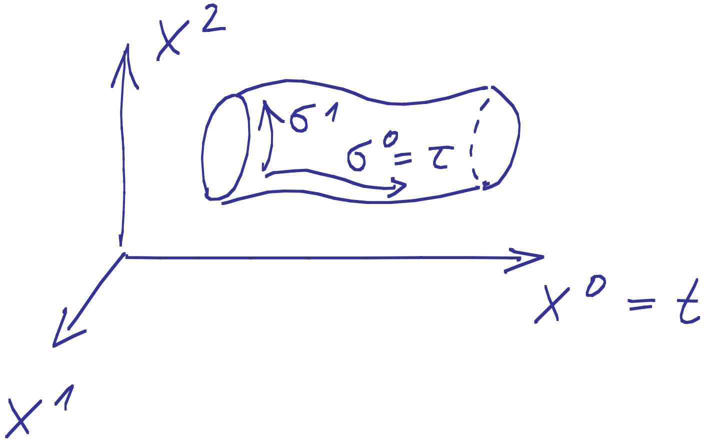


But this is equivalent to the "thin" torus we started from under $x^1 \leftrightarrow x^2$.

- More detail: Polchinski, Comm. Math. Phys. <u>104</u> ('86) 37

  Less detail: "Strings are a smeared out version ⇒ no UV divergence."

- <u>Dynamics in more detail:</u>



⇒ 2d QFT with

$$S \sim \frac{1}{\ell_s^2} \int d^2\sigma \sqrt{-h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$$

WS-metric

TS-metric

(This "Polyakov action" is classically equivalent to the "Nambu-Goto action" $S \sim$ surface area $/ \ell_s^2$.)

- D-dim.-TS-particles = States of this 2d-QFT (in fact CFT) on $\mathbb{R} \times S^1$ (i.e. vacuum, 1st excitation, 2nd exc. ...)

- D-dim. Poinc. symm. = internal symm. of WS-theory. We need it to be anomaly-free ⇒ $D = 26$ ("critical dimension")

- ⊕ : Graviton (+ other fields) in TS theory.
  ⊖ : No fermions ; Tachyon (state with $m^2 < 0$).

- Proposed resolution: Supersymmetrize WS-CFT :

$$X^\mu \Leftrightarrow \psi^\mu \qquad \Rightarrow \quad D_{crit.} = 10$$

(D 2d-spinors)

- Actually: $X_+^\mu \Leftrightarrow \psi_+^\mu$; $X_-^\mu \Leftrightarrow \psi_-^\mu$ (left- & right movers)
  ⇒ 2 SUSY's (Both in 2d and 10d)

- Two different consistent choices of spinor boundary conditions (on $\mathbb{R} \times S^1$ and other surfaces) ⇒ Two 10d theories

- Type IIB / IIA (same / opposite handedness of 10d SUSY generators)
- Can gauge resulting $\mathbb{Z}_2$ - symm. of IIB ⇒ Type I
- Can combine bosonic "+" side with superstring "−" side in two* different ways ⇒ heterotic SO_{32} & heterotic $E_8 \times E_8$

*) These are the only groups of rank 16 with even, self-dual root lattices.

$$\boxed{\Rightarrow \text{ 5 consistent matching* all known 10d SUGRAs}}$$

(*) Spectrum & all vertice which have been checked)

Our Example: IIB

$$S = \frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-g}\,\Big\{e^{-2\varphi}\Big(R+4(\partial\varphi)^2-\frac{1}{2}H_3^2\Big)-\frac{1}{2}\Big(F_1^2+F_3^2+F_5^2\Big)$$

$$+\dots\Big\}$$

1) fermionic terms

2) "CS-type" terms resulting e.g.
from $F_5 \to \tilde{F}_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$

Note: $H_3 = dB_2$; $F_1 = dC_0$; $F_3 = dC_2$; $F_5 = dC_4$

These are <u>form-fields</u> (& their field strengths):

$$F_1 = dC_0 = (\partial_\mu C_0)\,dx^\mu \quad ; \quad F_1^2 \sim (\partial_\mu C_0)(\partial^\mu C_0) = (\partial C_0)^2$$

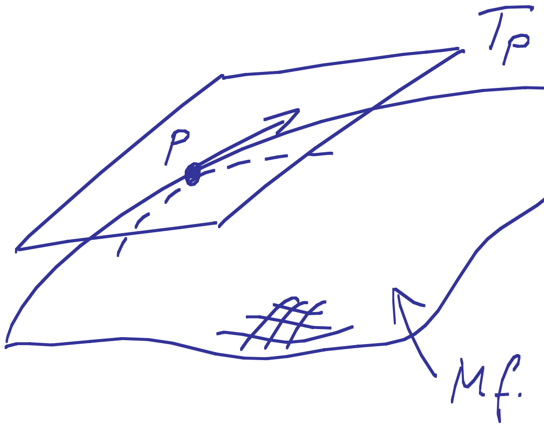
[just as for φ]

$$\mathrm{I\!I}A\begin{cases} F_2 = dC_1 \sim d\big((C_1)_\mu dx^\mu\big) \sim \partial_\nu (C_1)_\mu\, dx^\nu\wedge dx^\mu \sim (F_2)_{\mu\nu}\, dx^\mu\wedge dx^\nu \\ F_2 \sim (F_2)_{\mu\nu}(F_2)^{\mu\nu} \quad \text{as in ED} \end{cases}$$

$$F_3 = dC_2 \sim d\left((C_2)_{\mu\nu}\, dx^\mu \wedge dx^\nu\right) \sim \partial_\rho (C_2)_{\mu\nu}\, dx^\rho \wedge dx^\mu \wedge dx^\nu$$

etc.

An aside on diff. forms



basis: $\partial_\mu = \frac{\partial}{\partial x^\mu}$

T_p^*-basis: dx^μ

$$\left[\, \partial_\mu f = (\partial_\mu)\cdot(df) = (\partial_\mu)\cdot\left((\partial_\nu f)\, dx^\nu\right) = \delta_\mu{}^\nu \partial_\nu f = \partial_\mu f \,\right]$$

$dx^{\mu_1} \otimes \cdots \otimes dx^{\mu_p}$ – basis of $(T_p^*)^{\otimes p}$

- Total antisymmetrization:

$dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p}$ – basis of $(T_p^*)^{\wedge p}$

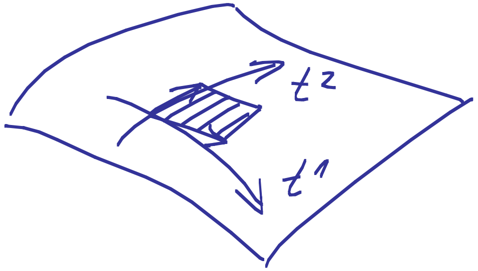
Diff. p-forms $A_p \sim (A_p)_{\mu_1 \cdots \mu_p}\, dx^{\mu_1} \cdots dx^{\mu_p}$ are the natural objects to integrate over p-dim. submanifolds.

- A_0 ; point $C_0 = x$ $\Rightarrow$ $A_0(x)$
- A_1 ; curve C_1 $\Rightarrow$ $\int_{C_1} A_1 = \int_{C_1} dt \left[\left(\frac{dx^\mu}{dt}\right)\cdot \partial_\mu\right]\cdot\left[(A_1)_\nu dx^\nu\right]$

$$= \int_{C_1} dt \, (A_1)_\mu \frac{dx^\mu}{dt} \quad \text{"="} \quad S[\text{electron}]$$

- A_p ; hypersurface C_p $\Rightarrow$ $\int_{C_p} dt^1 \cdots dt^p \, (A_p)_{\mu_1 \cdots \mu_p} \left(\frac{\partial x^{\mu_1}}{\partial t^1}\right) \cdots \left(\frac{\partial x^{\mu_p}}{\partial t^p}\right)$

$p = 2$:



$$dt^1 \cdots dt^p = dt^{1'} \cdots dt^{p'} \det\left(\frac{\partial t}{\partial t'}\right)$$

$$A_{\mu_1 \cdots \mu_p} \left(\frac{\partial x^{\mu_1}}{\partial t^1}\right) \cdots \left(\frac{\partial x^{\mu_p}}{\partial t^p}\right) = A_{\mu_1 \cdots \mu_p} \left(\frac{\partial x^{\mu_1}}{\partial t^{\nu_1'}}\right) \cdots \left(\frac{\partial x^{\mu_p}}{\partial t^{\nu_p'}}\right) \cdot \underbrace{\left(\frac{\partial t^{\nu_1'}}{\partial t^1}\right) \cdots \left(\frac{\partial t^{\nu_p'}}{\partial t^p}\right)}_{\text{because of total antisymm. of } A\text{, we can antisymmetrize this!}}$$

$$\Rightarrow \qquad \left(\frac{\partial t^{\nu_1'}}{\partial t^1}\right)\cdots\left(\frac{\partial t^{\nu_p'}}{\partial t^p}\right) \longrightarrow \frac{1}{p!}\,\varepsilon^{\nu_1\cdots\nu_p}\,\det\left(\frac{\partial t'}{\partial t}\right)$$

$\Rightarrow$ The determinants cancel and we get

$$\int_{C_p} dt^1 \cdot\cdot\, dt^p\,(A_p)_{\mu_1\cdot\cdot\mu_p}\left(\frac{\partial x^{\mu_1}}{\partial t^1}\right)\cdot\cdot\left(\frac{\partial x^{\mu_p}}{\partial t^p}\right) = \int_{C_p} dt^{1'} \cdot\cdot\, dt^{p'}\,(A_p)_{\mu_1\cdot\cdot\mu_p}\left(\frac{\partial x^{\mu_1}}{\partial t^{1'}}\right)\cdot\cdot\left(\frac{\partial x^{\mu_p}}{\partial t^{p'}}\right)$$

$\Rightarrow$ Integral reparametrization invariant and hence well-defined!

<u>"Corollary"</u>: C_2, C_4 → D1, D3 - branes as sources

(like electron for A_1)

[Also: $F_3 = dC_2$; $F_7 = * F_3 \sim \sqrt{-g}\,\varepsilon_{\mu_1\cdots\mu_7 \, 89\tau}\,(F_3)^{89\tau}\,dx^{\mu_1}\wedge\cdot\cdot\wedge dx^{\mu_7}$

$\uparrow$ Hodge operator

- $F_7 = dC_6 \quad \rightarrow \quad$ D5 - brane

  $C_0 \dashrightarrow C_8 \quad \rightarrow \quad$ D7 - brane

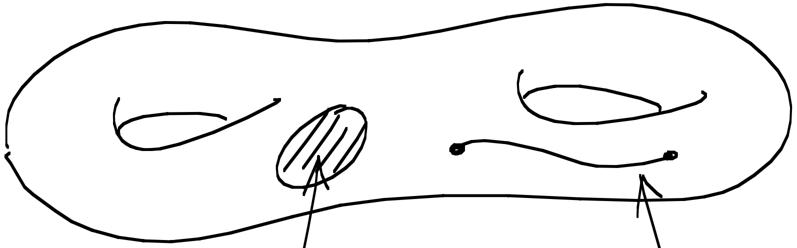
- Thus, in IIB we have Dp-branes with p odd.

  (analogously: IIA → p even)

  [Consistent with WS perspective, where branes are objects on which strings can end.]

An aside on homology / cohomology

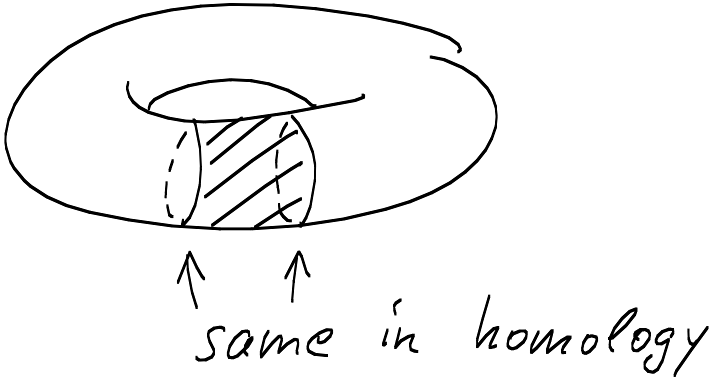
- consider (linear combinations of) submanifolds: chains



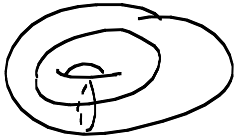
- Boundary operator : $\partial_p : C_p \to C_{p-1}$; $(\partial_p)^2 = 0$
- p-homology : $H_p := \ker \partial_p / \operatorname{Im} \partial_{p+1}$

e.g. $H_1 :=$ curves w/o bd. / curves which are bds.

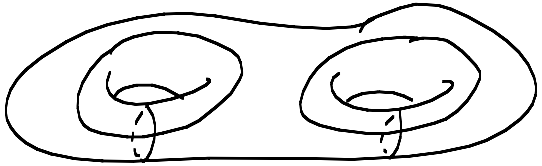
In other words: curves differing by a boundary are identified in homology:



- <u>Hence</u>: $\dim H_1(T^2) = 2$



$\dim H_1\Big($  $\Big) = 4$



- The "intersection product" provides a natural scalar product between H_p and H_{d-p} (e.g. H_1 with itself if $d = 2$)

- The "dual" concept of cohomology is based on $d_p : A_p \mapsto A_{p+1}$

$$d(A_{\mu_1 \cdots \mu_p} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_p}) = (\partial_{\mu_0} A_{\mu_1 \cdots \mu_p}) dx^{\mu_0} \wedge \cdots \wedge dx^{\mu_p}$$

(e.g. $df = (\partial_\mu f) dx^\mu$)

- Crucial fact: $d^2 = 0$
- Def.: $H^p := \ker d_p / \mathrm{Im}\, d_{p-1} \equiv$ closed forms / exact forms
- Important: H_{d-p} is dual to H_p (intersection product)

  H^p is dual to H_p ($\int_{C_p} A_p$)

  $\Rightarrow \underline{H_{d-p} \sim H^p}$ (Poincare duality)

- Also: $\int_M A_p \wedge A_{d-p}$ provides the "dual version" of the intersection product

- To demonstrate that all of this is independent of the representative one chooses in a (co)homology class, one needs Stokes' theorem:

$$\int_{C_p} dA_{p-1} = \int_{\partial C_p} A_{p-1}$$

- Given a metric, each coh. class has one natural representative: a (unique!) harmonic form:

$$dA_p = 0 \quad \& \quad d{*}A_p = 0$$

$$\Big(\text{or } A_p \in \ker \underset{\substack{\uparrow \\ \text{Laplace operator}}}{\Delta} \;;\; \Delta = *d*d + d*d* \Big)$$

- Returning to physics:

Next step: $\underline{\underline{10d \to 4d}}$, i.e. $S = \frac{1}{2\kappa_{10}^2} \int_{\mathbb{R}^4 \times M_6} d^{10}x \sqrt{-g} \left\{ e^{-2\varphi} R + \cdots \right\}$

Toy model: 5d → 4d on $\mathbb{R}^4 \times S^1$

$$S = \int d^5x \, \tfrac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) \quad ; \quad \mu = 0, \cdots, 3, 5$$

vacuum solution: $\varphi \equiv 0$; we expand around this

rename $x^5 \to y$

Ansatz: $\varphi(x,y) = \sum_{n=0}^{\infty} \varphi_n^c(x) \cos(ny/R) + \sum_{n=1}^{\infty} \varphi_n^s \sin(ny/R)$

$$\Rightarrow S = \frac{2\pi R}{2} \int d^4x \left[(\partial \varphi_0^c)^2 + \frac{1}{2} \sum_{n=1}^{\infty} \left\{ (\partial \varphi_n^c)^2 + m_n^2 (\varphi_n^c)^2 + (\partial \varphi_n^s)^2 + m_n^2 (\varphi_n^s)^2 \right\} \right]$$

with $m_n = n/R$

⇒ 4d theory of zero-mode + KK modes

Lessons: - We started from class. solution $\varphi \equiv 0$

- Our zero-mode parametrizes the degeneracy of vacuum ($\varphi \equiv c$ is ok for any c). ⇒ Moduli space

– Our zero-mode is solution of $\Box_{S_1} \varphi(y) \sim (\partial_y)^2 \varphi(y) = 0$

The "real thing": $10d \to 4d$; $S = S_{grav.} + \cdots$

$$\Downarrow$$

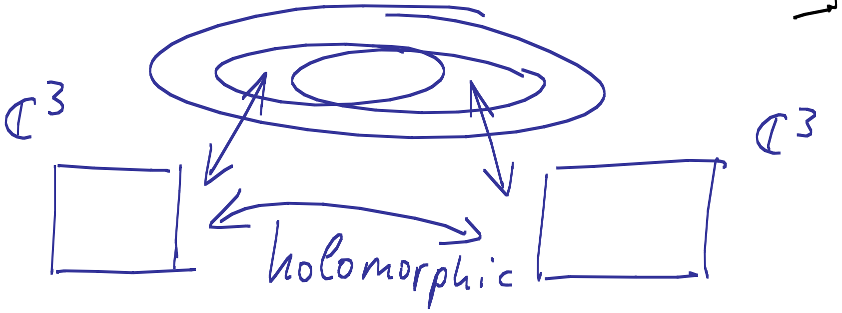
M_6 being Ricci-flat ($R_{\mu\nu} = 0$) should provide a good starting point.

- A large class of such M_6 is provided by CY_3
- CY_3 : 3-compl.-dim. Kähler manifold with vanishing 1st Chern class.
- Crucial: CYs have (a moduli space of) Ricci flat metrics

  [$\exists\, g_{\mu\nu}(y)$ ($\mu = 1 \cdots 6$) with $R_{\mu\nu}[g] = 0$;

  Think of T^6 for now, but this is in fact too special.]
- In more detail:

  compl. manifold: $\exists$ atlas with



- Holomorphicity of coordinate change:

$$z^{i'} \underset{\uparrow \text{general}}{=} z^{i'}(z^1, z^2, z^3, \bar z^1, \bar z^2, \bar z^3) \underset{\uparrow \text{holomorphic}}{=} z^{i'}(z^1, z^2, z^3)$$

- Kähler: $ds^2 = g_{i\bar j}\, dz^i d\bar z^{\bar j}$; $\underbrace{g_{i\bar j} = \frac{\partial}{\partial z^i}\frac{\partial}{\partial \bar z^{\bar j}} \underbrace{K(z,\bar z)}_{\substack{\text{real fct.} \\ \text{(Kähler potential)}}}}_{\substack{\text{locally} \\ \text{(in each patch)}}}$

$$\underline{J = i g_{i\bar j}\, dz^i \wedge d\bar z^{\bar j}}$$

(Kähler form)

(cf. "complex structure" $J_i{}^j$)

- 1st Chern class vanishes ($c_1 = 0$): $\Leftrightarrow$ $(\text{holom. tangent Bundle})^{\wedge 3}$ is trivial
- <u>This is equivalent to:</u> $\exists$ (unique) nowhere vanishing holom. 3-form

$$\Omega = f(z^1, z^2, z^3)\, dz^1 \wedge dz^2 \wedge dz^3 \quad \text{(locally).}$$

- J & Ω are a 2- and 3 form respectively.

  More precisely, a $(1,1)$-form and a $(3,0)$-form (cf. dz & $d\bar{z}$).

  One could, starting from the "de Rham" cohomology (above) develop the Dolbeault coh. (e.g. $H^3 \rightarrow H^{(3,0)} \oplus H^{(2,1)} \oplus H^{(1,2)} \oplus H^{(0,3)}$). We have no time for this.

- Now comes the crucial step of dim. reduction of S_{IIB} on a CY_3. Our interest in the (metric) moduli:

$$\int_{10} \sqrt{-g}\, R[\underset{\uparrow}{g}] \longrightarrow \int_{10} \sqrt{-g}\, R[g + \underset{\uparrow}{\delta g}]$$

some Ricci-flat metric   another Ricci-flat metric

$$\delta g = f^\alpha(x)\, \delta g^\alpha(y)$$

$$\ldots = \int_4 \sqrt{-g_4} \left\{ (\partial_\mu f^\alpha)(\overline{\partial^\mu f^\beta})\, G_{\alpha\bar\beta}[f] + \cdots \right\}$$

- The 4d fields f^α are the (complex) moduli
- $G_{\alpha\bar\beta}[f]$ is the metric on the moduli space (the f-dependence arises only at a finite distance in the expansion around our orig $g(y)$).
- Two types of deformation:

1) $\delta g_{i\bar j} = -i\,\delta J_{i\bar j}$ (Kähler def.)

2) $\delta g_{\bar i\bar j} = -\frac{1}{\|\Omega\|}\,\bar\Omega_{\bar i}{}^{kl}\,\delta\chi_{kl\bar j}$; $\delta\Omega = \underset{(3,0)}{(\delta\epsilon)\cdot\Omega} + \underset{(2,1)}{\delta\chi}$

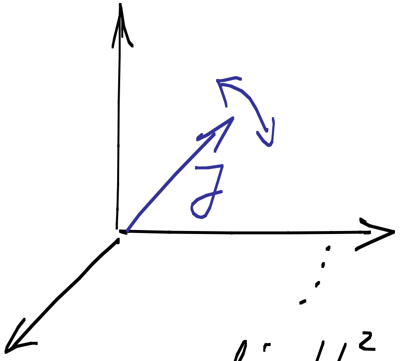
(Compl. structure def.)

- Crucial for us:

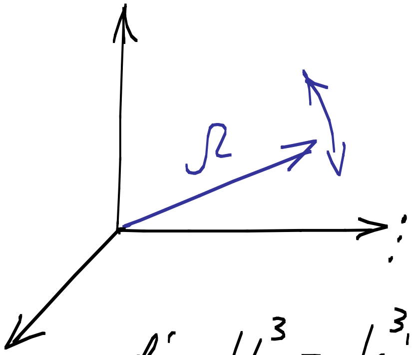
Motion in mod. space can be understood as motion of J & Ω (as cohomology classes) in H^2 and H^3 respectively.

(Calabi: Given J, Ω, the Ricci flat metric is unique.
Yau: Proof of existence)

- Thus, even though we do not know $g_{\mu\nu}(y)$ (at any point in mod. space) explicitly, all we need for the 4d eff. theory follows from:



$$\dim H^2 = h^{1,1}$$



$$\dim H^3 = \underbrace{h^{3,0}}_{1} + \underbrace{h^{2,1} + h^{1,2}}_{2h^{2,1}} + \underbrace{h^{0,3}}_{1}$$

- <u>More explicitly:</u>

  (maybe) simplest "real" CY_3: <u>Quintic</u>

- $\mathbb{C}P_4 = \mathbb{C}^5 \setminus \{0\} / \sim$ where $(z^1, \dots, z^5) \sim (\lambda z^1, \dots, \lambda z^5)$

  with any $\lambda \in \mathbb{C} \setminus \{0\}$

- <u>Quintic</u>: submanifold def. by

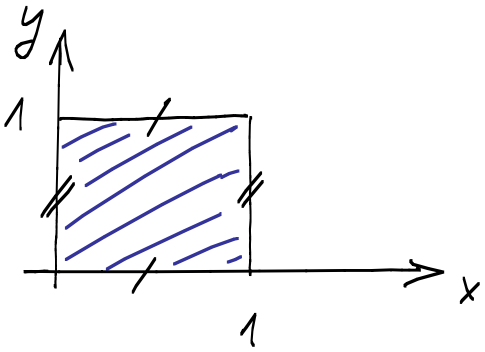
$$P^5(z^1, \dots, z^5) = \alpha \cdot (z^1)^5 + \beta \cdot (z^1)^4 z^2 + \dots + \gamma (z^5)^5 \overset{!}{=} 0$$

- This is too complicated for now (see however, e.g.,
  GSW, vol. II ; Candelas: lectures on compl. mfs. ;
  B. Greene - hep-th/9702155 ; T. Hübsch: CY manifolds...

- Our toy model: T^2 (the one and only "CY_1")

T^2 as real mf.: $\mathbb{R}^2/\sim$ where $(x,y) \sim (x+n, y+m)$

for any $(n,m) \in \mathbb{Z}^2$.



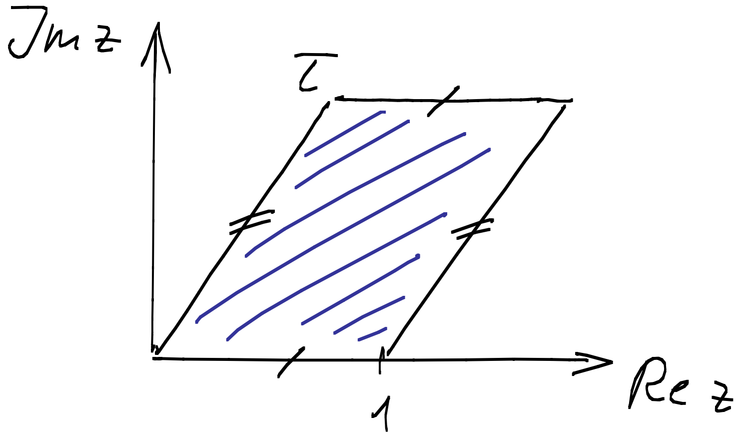
$x, y \in [0,1)$

$$ds^2 = g_{xx}\,dx^2 + 2g_{xy}\,dx\,dy + g_{yy}\,dy^2$$

T^2 as compl. mf.: $\mathbb{C}/\sim$ where $z \sim z + n + m\tau$

for any $(n,m) \in \mathbb{Z}^2$

($\tau \in \mathbb{C}$ fixed!)



$$z = x + \tau y$$

- hol. 1-form: $\Omega = \alpha\, dz = \alpha\, dx + \alpha \tau\, dy$
- the position of Ω in H^1 (actually, in its complexification) can be characterized by its "periods": $\Pi_i = \int_{C_{1,i}} \Omega$

$$\Pi_1 = \int_{y = \text{const.}} \Omega = \int_0^1 \alpha\, dx = \alpha$$

$$\Pi_2 = \int_{x = \text{const.}} \Omega = \int_0^1 \alpha \tau\, dy = \alpha \tau$$

$\Rightarrow$ "period vector" $\Pi = (\tau_1, \tau_2)$

$$\tau = \frac{\Pi_2}{\Pi_1}$$

- Kähler form: $J = \underset{\substack{\uparrow \\ \text{real coeffs.}}}{t^i} \; \underset{\substack{\nwarrow \\ \text{basis of harmonic 2-forms } (\hat{=} \text{ 4-cycles in } CY_3)}}{\omega_i}$

For T^2, there is just one basis element. Hence

$$J = i g_{z\bar{z}} \, dz \wedge d\bar{z} = i g_{z\bar{z}} \left(dx \wedge \bar{\tau} dy + \tau dy \wedge dx \right)$$

$$= \underbrace{-i(\tau - \bar{\tau}) \; g_{z\bar{z}}}_{\text{"}t\text{"}} \cdot dx \wedge dy$$

- We can now return to our original metric:

$$ds^2 = g_{z\bar{z}} \, dz d\bar{z} = g_{z\bar{z}} \left(dx^2 + |\tau|^2 dy^2 + (\tau + \bar{\tau}) dx dy \right)$$

$$\Rightarrow \quad g_{\mu\nu} = g_{z\bar{z}} \begin{pmatrix} 1 & \mathrm{Re}\,\tau \\ \mathrm{Re}\,\tau & |\tau|^2 \end{pmatrix} = \frac{t}{2\, \mathrm{Im}(\Pi_2/\Pi_1)} \begin{pmatrix} 1 & \mathrm{Re}\, \Pi_2/\Pi_1 \\ \mathrm{Re}\, \Pi_2/\Pi_1 & |\Pi_2/\Pi_1|^2 \end{pmatrix}$$

$\Rightarrow$ We have expressed $g_{\mu\nu}$ in terms of the pos. of (the cohom. classes of) J & Ω in H^2 & H^1.

(cf. book by Becker/Becker/Schwarz ;
App. of hep-th/0505260 by Antoniadis et al.)

This also works for real CY_3, but not explicitly. Though we can't get $g_{\mu\nu}(y)$, we can get the coeffs. of the kinetic terms of moduli in the 4d eff. theory.

- We now return to the "real thing", ST on CY_3. We know a priori that we will get 4d SUSY. (Alt. def. of CY: M_6 with covar. const. spinor;
Yet another def: M_6 with SU_3 holonomy)
- In fact: IIB $\Rightarrow$ $N=2$ SUSY in 4d.

[We will display only $N=1$ since, in the end, we care about "CY orientifolds", i.e. $(CY_3/\mathbb{Z}_2)$. This is needed for D7-branes, needed for pheno.

- Hence: $$\mathcal{L}_{4d} = K_{i\bar{j}}\,(\partial x^i)(\overline{\partial x^j}) - V(x,\bar{x}) + \text{gauge} + \text{fermions} + \text{other fields}$$

↑ Kähler metric on moduli space

$$V = e^K\left(K^{i\bar{j}}(D_iW)(\overline{D_jW}) - 3|W|^2\right) \quad ; \quad D_iW \equiv \partial_iW + K_iW$$

$$i,j = 1,\dots,h^{1,1} + h^{2,1} + 1$$

↑ # of Kähler moduli

↑ # of compl. structure moduli

↑ axio-dilaton

(I) <u>Kähler:</u> $J = t^\alpha \omega_\alpha \quad ; \quad \alpha = 1\cdots h^{1,1}$

$$\text{Vol.} = \frac{1}{6}\int J^3 = \frac{1}{6}\,\kappa_{\alpha\beta\gamma}\,t^\alpha t^\beta t^\gamma$$

↑ 2-cycle volumes

$$\tau_\alpha = \kappa_{\alpha\beta\gamma}\,t^\beta t^\gamma \quad \leftarrow \text{4-cycle volumes}$$

$K_k = -2 \ln \text{Vol.}$; $\text{Vol.} = \frac{1}{6} \kappa_{\alpha\beta\gamma} t^\alpha t^\beta t^\gamma$;

$$t^\alpha = t^\alpha(\tau_1 + \bar{\tau}_1, \ldots, \tau_{h^{1,1}} + \bar{\tau}_{h^{1,1}})$$

↑

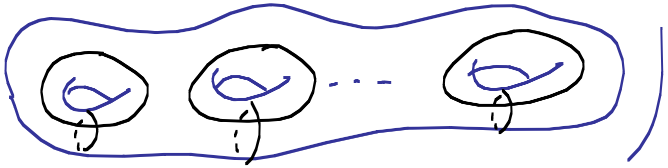
proper (compl.) fields of $N=1$ SUSY

(also redefined by e^φ ...)

(II) Compl. Structure: Choose (symplectic) basis of 3-cycles A^a, B_b :

$A^a \cdot A^b = 0$, $B^a \cdot B^b = 0$, $A^a \cdot B_b = \delta^a{}_b$

(recall:



)

Introduce periods as $z^a = \int_{A^a} \Omega$; $\mathcal{G}_b = \int_{B^b} \Omega$

$\Pi = (z^a, \mathcal{G}_b(z^1, \ldots, z^{h^{2,1}}))$ (period vector)

$$K_{cs} = -\ln\left(i\int \Omega\wedge\bar{\Omega}\right) = -\ln\left(-i\,\Pi^{+}\underset{\uparrow}{\Sigma}\,\Pi\right)$$

symplectic metric $\sim \begin{pmatrix} 0 & 1\!\!1 \\ -1\!\!1 & 0 \end{pmatrix}$

$$= -\ln\left(-i\,\bar{z}^{a}\mathcal{G}_{a}(z) + i\,z^{a}\,\overline{\mathcal{G}_{a}(z)}\right)$$

Together: $K = K_{K} + K_{cs} - \ln\left(-i(\phi-\bar{\phi})\right)$; $\phi = C_0 + i e^{-\varphi}$

$(M_4 = \alpha' = 1)$

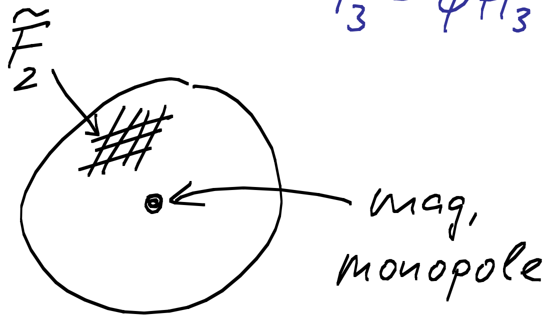
$\left[\text{Also: } -\ln(S+\bar{S}) \; ; \; S = -i\phi\right]$

(cf. Polchinski, Quevedo, Douglas/Kachru, ... as cited earlier)

Flux:

$$W = \int \underbrace{G_3}_{F_3 - \phi H_3} \wedge\,\Omega = (2\pi)^2\,(\underset{\uparrow}{f} - \phi\,\underset{\uparrow}{h})\cdot\Pi(z)$$

integer valued vectors



(for quantization of F_3, H_3 on 3-cycles, cf. quantization of $\tilde{F}_2$ of ED on S^2)

- Now let's first ignore the τ's, call $\phi \equiv z^0$ and consider $W = W(z)$ as induced by flux.

  Note: typical CYs have $O(100)$ 3-cycles ; let $|f_a|, |h_a| \lesssim 5$

  $\Rightarrow (10 \times 10)^{100} = 10^{200}$ flux choices

  (compare: # of atoms in universe $\sim 10^{80}$!)

- Fact: $V(z,\bar{z})$ has minimum (with $V_{min} < 0$) if $\underline{\underline{D_i W = 0 \text{ for all } z^i}}$

  $\Rightarrow h^{2,1}+1$ equs. with $h^{2,1}+1$ variables $\Rightarrow$ sol. generically exists

  $$\Rightarrow V = e^{K}\Big(\underbrace{|DW|^2}_{=0} - 3|W|^2\Big) = -e^{-K} \cdot 3|W|^2$$

  "SUSY AdS vacua"

  But this is not true because of τ's :

- Now remember about the τ's : W is indep. of τ's

(no F_2/F_4 in type IIB)

- Treat z's as fixed by $D_i W = 0 \Rightarrow W = W_0$

$$\Rightarrow V = e^K \left(|D_\alpha W_0|^2 - 3|W_0|^2 \right)$$

$$= e^{-2\ln(\mathrm{Vol.})} \Big(\underbrace{K^{\alpha\bar\beta} K_\alpha K_{\bar\beta}}_{\equiv 3} - 3 \Big) |W_0|^2$$

if e^K is homog. fct. of degree -3 in the variables $(\tau^\alpha + \bar\tau^{\bar\alpha})$

[as is indeed the case for the classical moduli space of Kähler moduli]

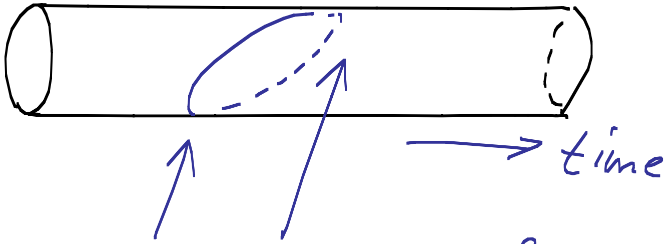
[Exercise: Check this for $K = -3\ln(T+\bar T)$ and then generally.]

$\Rightarrow \quad V \equiv 0$ "No scale model"

- Now include non-pert. corrections: $W = W_0 + e^{-T}$ (just one T for now)

Origin: e.g. D3-instantons wrapping 4-cycle.

Example: D0-instanton wrapping 1-cycle



Spont. pair creation & annihilation

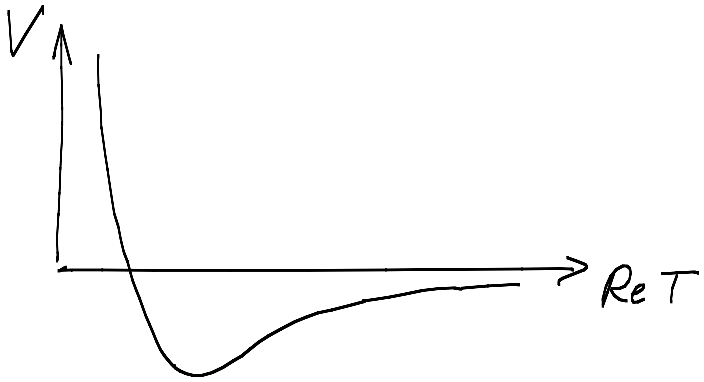
- Now $V \neq 0$. We need to calculate T for which $D_T W = 0$

$$\Rightarrow \quad -e^{-T} - \frac{3}{T+\bar{T}}\left(W_0 + e^{-T}\right) \overset{!}{=} 0$$

Let T be real and rewrite: $W_0 = -\left(1 + \frac{2}{3}T\right)e^{-T}$

- Hence, choosing $W_0 < 0$ and $|W_0| \ll 1$, we find a solution at parametrically large volume: $T \sim \ln(1/|W_0|)$.

- The potential is easy to analyse qualitatively



SUSY-AdS vacuum.

- Thus, all moduli stabilized by C.C. is neg. & $\sim |W_0|^2 \sim \frac{e^{-2T}}{T}$

- Resolution: CYs w/ fluxes actually have strongly warped regions

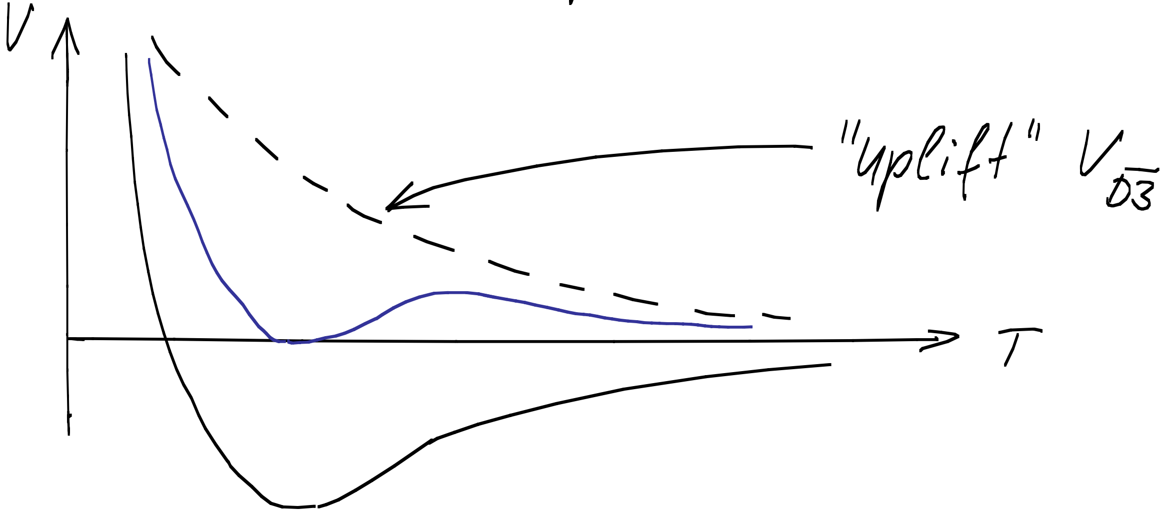
(cf. RS models)

"KS throat"

(here local (stringy) energy scales are expon. small from 4d perspective)

place $\overline{D3}$-brane (breaking SUSY explicitly) at tip of throat

$$\Rightarrow \quad V_{\overline{D3}} \sim \frac{\text{small \#}}{T^2} \overset{!}{\sim} \frac{e^{-2T}}{T}$$



- Some of the essential original papers are:

  Gukov, Vafa, Witten '99 ; Dasgupta, Rajesh, Sethi '99, GKP, KKLT

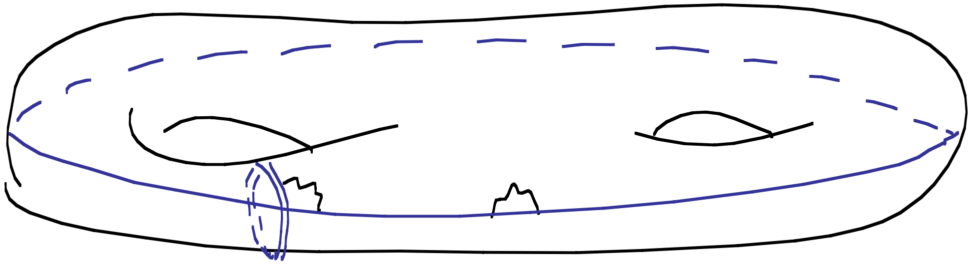
- An alternative stabilization scenario for τ's is the "LARGE volume scenario" of Balasubramanian, Berglund, Conlon, Quevedo '05 (which also uses perturbative corrections for τ's)

- As a crucial result, the "landscape" has become "reality" (at least if ST is the correct UV completion of quantum gravity).

- To also get the SM, more ingredients are obviously needed. In IIB, one natural choice are D7 branes (in IIA: D6)

- Toy model picture:

  $(SU_2 \times U_1)$



- In IIB, D7 branes wrap holomorphic submanifolds of codim. = 1, i.e. <u>divisors</u> (this is just as we defined the quintic in $\mathbb{C}P^4$).

- Non.-pert. regime ($g_s \sim 1$): New "branes" with strong backreaction and with exceptional groups (E_6, E_7, E_8) are possible.

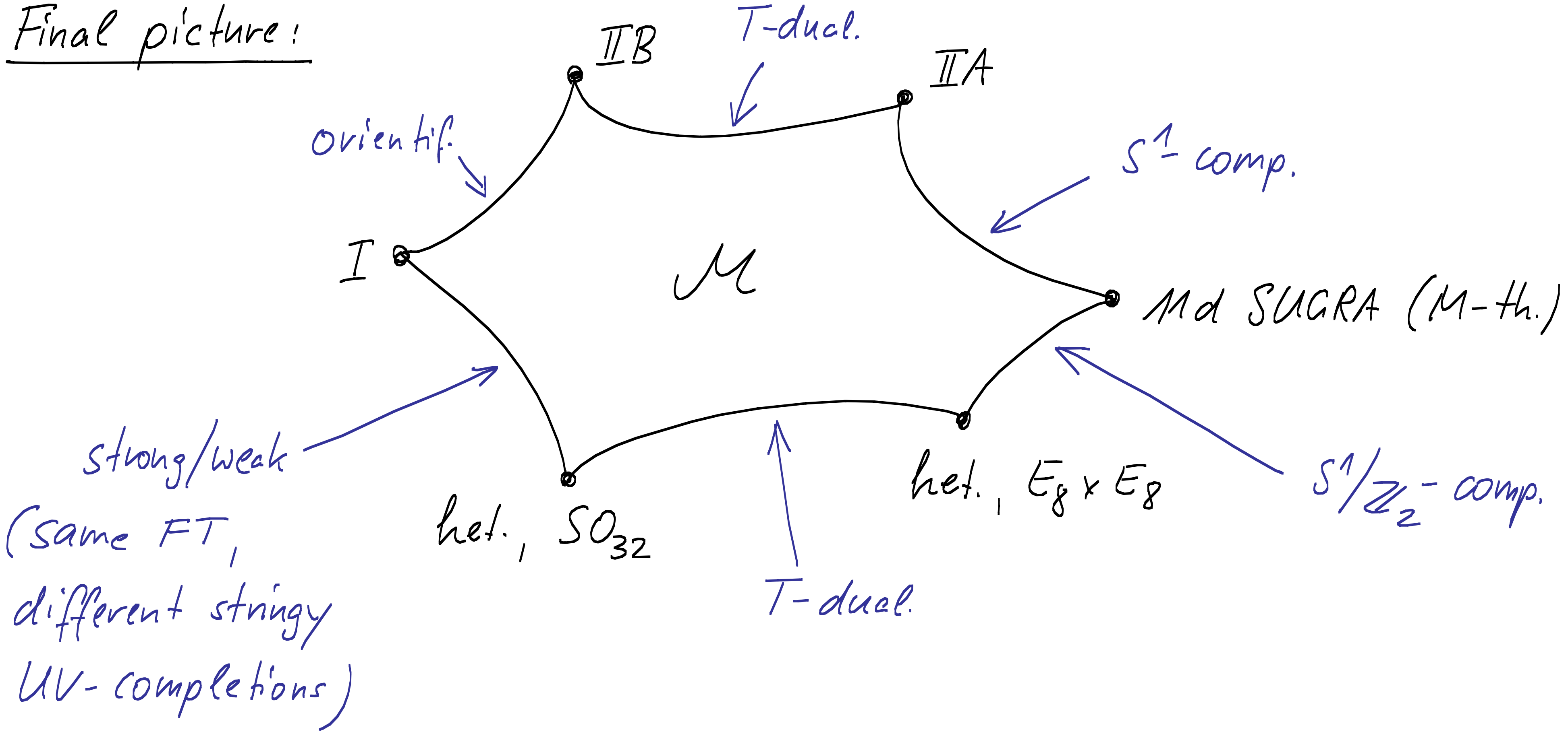
- This is relevant for GUT model building (Beasley, Heckman, Vafa; Donagi, Wijnholt)

- Also: Large top Yukawa coupling in SU_5 GUT (broken by F_2-flux) is now possible.

  → "F-theory GUTs" (e.g. lectures by Weigand, 1009.3497)

- Another approach (in fact historically the main approach to string pheno) is het./type I on CY_3 or orbifolds. Here, one has a SYM theory already in 10d. ($E_8 \times E_8$ or SO_{32} broken by $\langle F_2 \rangle \neq 0$ in 10d)

Final picture:



At the moment, only some corners allow for a reasonably controlled description (and even this is strongly limited by our calculational ability). The landscape is almost certainly much larger and more diverse than what is known so far.