Geometrical Challenges for the String Landscape

(Arthur Hebecker, Heidelberg)

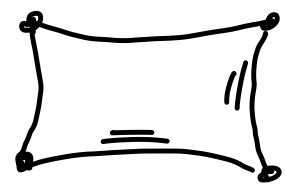
Plan:
- The landscape as accepted 2000 ~ 2018
- Problems discovered in the aftermath of the dS and other Swampland Conjectures
  - Singular-bulk problem of KKLT
  - Tadpole constraint of LVS
  - Tadpole conjecture as a potential show stopper for the whole Landscape

The landscape 2000 ... 2018

(Bousso/Polchinski, GKP, Denef/Douglas, KKLT, LVS)

ST $\longrightarrow$ 10d SUGRA on $\mathbb{R}^{1,3} \times X$; $\underbrace{X = CY/K}_{\text{"CY orientifold"}}$

(with K some finite group, e.g. $\mathbb{Z}_2$)

Vizualization: $T^2/\mathbb{Z}_2 =$ 

"orientifold planes"

(co-dimensions of these may vary)

$10d$ SUGRA $\supset$ $\underbrace{\text{metric field } g_{MN} \, ; \; B_{[MN]} \, ; \; \cdots}$

$\Downarrow$

4d effective field theory obtains certain field content, including in particular MODULI OF X

$\Downarrow$

$$\mathcal{L}_{4d} \supset \underbrace{K(z)_{i\bar{j}} (\partial z^i)(\partial \bar{z}^{\bar{j}})}_{\text{complex structure}} + \underbrace{K(T)_{\alpha\bar{\beta}} (\partial T^\alpha)(\partial \bar{T}^{\bar{\beta}})}_{\text{Kahler}} + \cdots$$

(complexified using integrals of p-form fields)

$\rightarrow$ Grimm / Louis
Jockers / Louis
Hosono / Klemm / Theisen / Yau ....

Kahler potentials for the Kahler metrics above:

$$K(z) = -\ln\left(\int_X \Omega \wedge \bar{\Omega}\right) \quad ; \quad \Omega = \text{holom. 3-form}$$

$$= -\ln\left(\Pi^\dagger \Sigma \Pi\right)$$

with periods $\Pi = \begin{pmatrix} 1 \\ z^1 \\ \vdots \\ z^n \\ \Pi^1(z) \\ \vdots \\ \Pi^n(z) \end{pmatrix}$; $n = h^{2,1}$

$$K(T) = -\ln\left(t^\alpha t^\beta t^\gamma \kappa_{\alpha\beta\gamma} + \cdots\right)$$

4-cycle volumes

with $\tau_\alpha = \kappa_{\alpha\beta\gamma} t^\beta t^\gamma$ and $\tau_\alpha = T_\alpha + \bar{T}_\alpha$

Moduli stabilization by fluxes

$$10d \text{ SUGRA} \supset F_3 - \underset{\uparrow}{S} H_3 \equiv G_3 \in \Omega^3(\mathbb{R}^{1,3} \times X^6)$$

$$C_0 + \frac{i}{g_s}$$

The "background flux" is quantized:

$$F_3, H_3 \in H^3(X, \mathbb{Z})$$

$\Rightarrow$ non-trivial scalar potential

$$V(z, \bar{z}) = K^{i\bar{J}} (D_i W) (\overline{D_j W}) \quad ; \quad D_i = \partial_i W + K_i W$$

$$W \sim \int_{X^6} G_3 \wedge \Omega$$

Explicitly: $\quad W \sim (f - Sh)\cdot\Sigma\cdot\Pi$

"flux vectors" built from $\int_{\Sigma_i} F_3/H_3$

The SUSY-condition ($\equiv$ vacuum condition)

$$D_i W = 0 \quad ; \quad D_S W = 0$$

"generically" stabilizes all moduli z^i, S

(n+1 eqs. for n+1 variables)

$\Longrightarrow$ as many vacua as choices of (f,h).

Tadpole

10d SUGRA $\supset F_5$

Sourced by flux & O-planes/branes

$$\int d * F_5 = \underbrace{\int F_3 \wedge H_3}_{=N} + \underbrace{\int j_{loc.}}_{\equiv Q} = 0$$

based on geometry of X

$(N = f \cdot \Sigma \cdot h)$

$$-Q = -\int j_{loc} = \frac{N_{O3}}{4} + \frac{\chi(O7)}{12} + \frac{\chi(O7) + \chi(O7')}{48}$$

Better: F-Theory ($\longrightarrow$ Vafa; Morrison; Weigand; ...)

$T_2 \to CY_4$; T_2 fibration encodes "S"; $-\int j_{loc} \triangleq \chi(CY_4)$

$\downarrow$

B_3

Key point:

Finiteness of available N $\Rightarrow$ Finiteness of landscape

(Denef/Douglas ... Grimm)

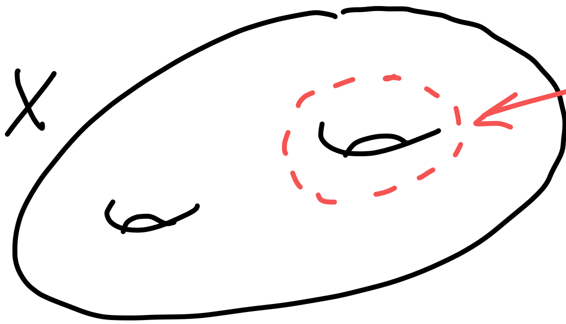
Also:

So far we only discussed complex-structure moduli & their stabilization.

Let us consider Kahler moduli next ...

"KKLT step 1": Kahler moduli stabilization

(assume that c.s.-moduli are integrated out)

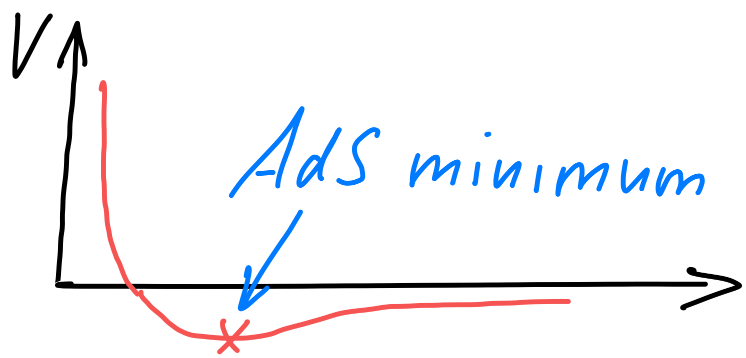


euclidean D3 brane wrapped on 4-cycle $\Rightarrow$ Instanton Correction

$$W = W_0 + \underline{e^{-T}}$$

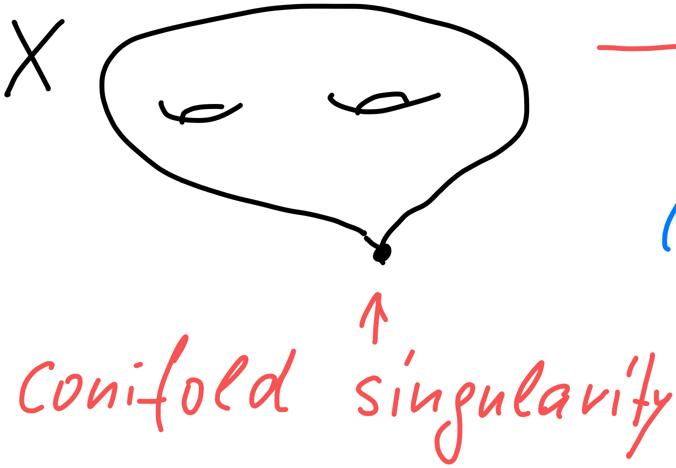
flux-effect

$$\boxed{V = e^{K}\left(|DW|^2 - 3|W|^2\right)}$$



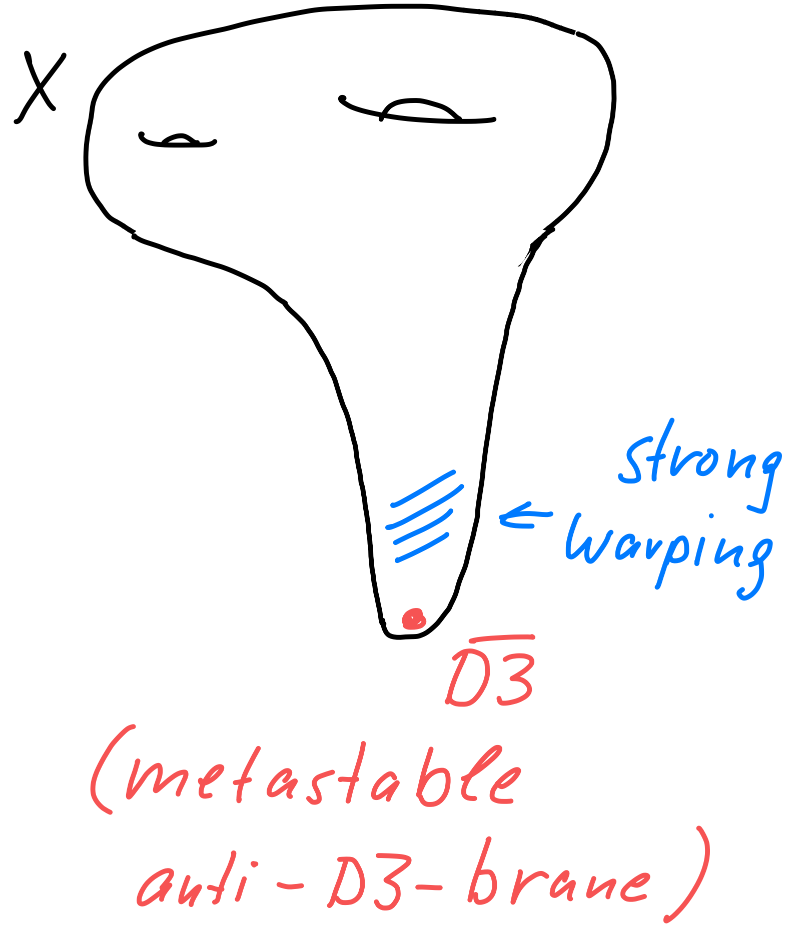
(need also $W_0 \ll 1$ by flux tuning)

"KKLT step 2": Uplift



flux

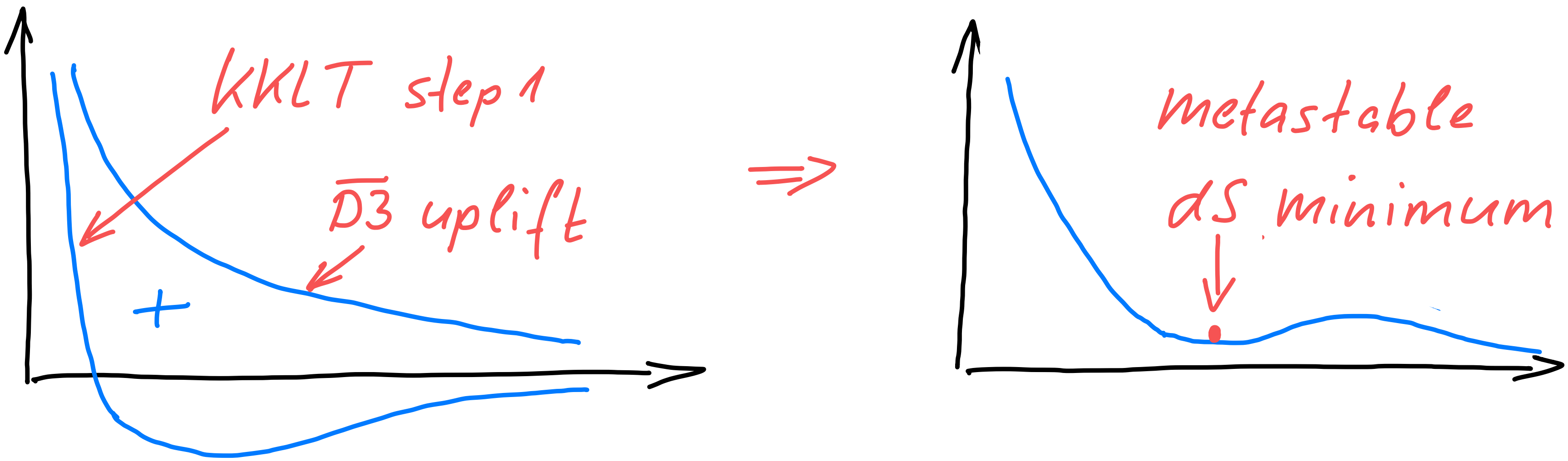
(→ warping)



$$\boxed{\begin{gathered} ds^2 = dx^2 + dy_{CY}^2 \\ \Downarrow \\ ds^2 = h^{-1}(y)\,dx^2 + h(y)\,dy_{CY}^2 \end{gathered}}$$

(Technical terms: Klebanov-Strassler-Throat;
KPV-uplift)

Uplift from AdS to dS minimum:



Unsatisfactory aspects:

- vacua with $W_0 \ll 1$ very hard to find explicitly
- anti-D3-uplift follows only from 10d EFT (no stringy or 4d SUSY derivation)

More recent developments

- This (and some important variants, like "LVS") has remained the main evidence for "stringy dS".
- It has been proposed that stringy dS is impossible as a matter of principle ("is in the Swampland").

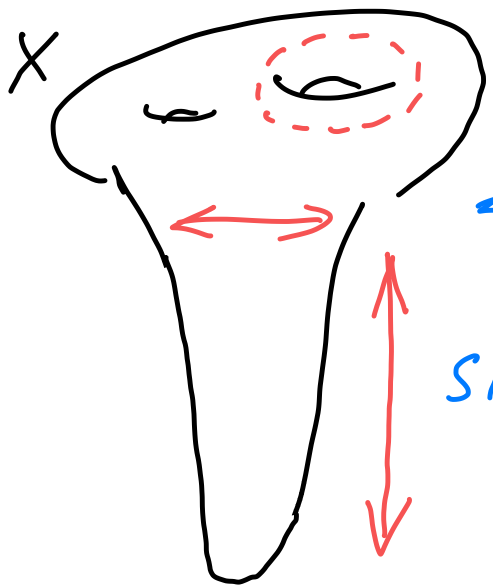
  [Danielsson/Van Riet ; Obied ⋯ Vafa '18]

  [see also: Bena, Grana, Sethi, Dvali, ⋯]
- Subsequently, proposals like KKLT & LVS have been subjected to intense scrutiny (with varying success)
- I will focus on what I feel is most critical ⋯

Singular bulk problem

[Carta/Moritz/Westphal '19 ; Gao/AH/Junghans '20]

[see however: Carta/Moritz ; McAllister et al. '21]

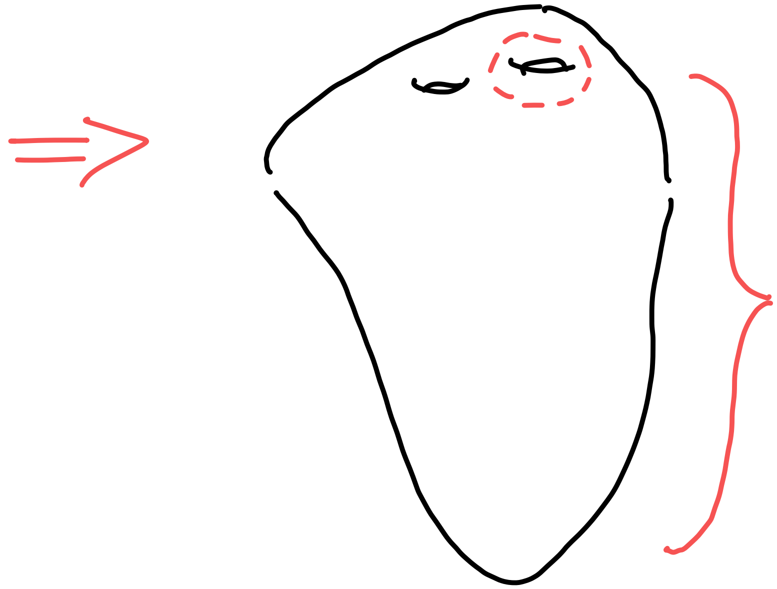


width of throat coupled to depth

$\Longrightarrow$ tends to become too wide for X

significant depth of throat needed to make uplift parametrically as small as AdS-minimum

$\Longrightarrow$ "exotic" geometry, with large throat & small bulk CY.



strong warping arises also in bulk CY region

$\Downarrow$

$h(y)$ goes to zero ; metric becomes undefined

$\Downarrow$

$K(T,\bar{T})$ not calculable ; parametric control lost

[Is a stringy understanding of singular bulk possible? Control of string-scale geometry?]

Technical Aside:

- Depth of AdS minimum $\sim e^{-2\,\mathrm{Re}(T)}$ ①
- Uplift potential $\sim e^{-N/g_s M^2}$ ②

total tadpole in KS throat

F_3 flux on 3-cycle of KS throat

- Metastable uplift needs ① $\sim$ ②

$\Rightarrow$ $\mathrm{Re}(T) \sim N/g_s M^2$

- Upper throat radius obeys $R^4 \sim N$
- Control at tip of throat: $g_s M^2 \gg 1$

$\lightning$!

possibly in better shape: Large Volume Scenario

[Balasubramanian/Berglund/Conlon/Quevedo '05] (or LVS)

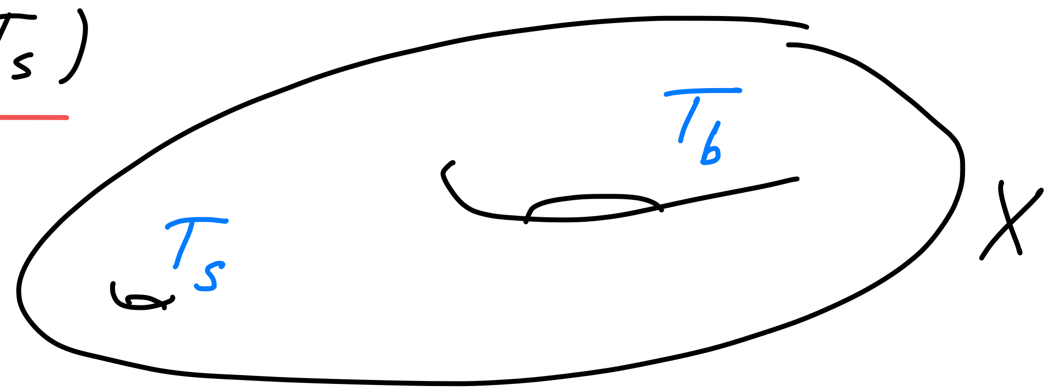
- generalizes KKLT by $T \to T_b, T_s$

with $K = -2\ln\left[(T_b+\bar{T}_b)^{3/2} - (T_s+\bar{T}_s)^{3/2} + \xi\right]$

and $W = W_0 + e^{-T_s}$

$\Rightarrow$ AdS minimum similar to KKLT, but with

$\mathrm{Re}(T_b) \gg \mathrm{Re}(T_s)$



more explicitly: $\mathcal{V} \equiv \mathrm{Vol}(X) \sim \mathrm{Re}(T_b)^{3/2} \sim \exp(1/g_s)$

to be tuned to small value

$\Rightarrow$ Thus, $\mathcal{V}$ is exponentially large, but this may still not be sufficient for control!

[Junghans '22]

Problem: Various higher-order corrections, e.g. related to

$$S \supset \int (R + R^2 + R^3 + \dots)$$

"higher curvature"

Our attempt to quantify the problem and identify the key obstruction: $\Rightarrow$

LVS Parametric Tadpole Constraint

[Cao / AH / Schreyer / Venken '22]

Volume large $\Rightarrow$ AdS minimum shallow $\left(\sim \frac{1}{\mathcal{V}^3}\right)$

$\Rightarrow$ Need throat to be deep $\Rightarrow$ need much flux in throat $(N_{th.} \gg 1)$

$\Rightarrow$ Curvature corrections of relative size $N_{th.} / \mathcal{V}^{2/3}$ represent a problem.

More explicitly: Since the uplift implies

$$\mathcal{V} \sim e^{O(1)\cdot N_{th.}} ,$$

one can be sure that for very large $N_{th.}$ corrections like $\boxed{N_{th.}/\mathcal{V}^{2/3}}$ will be small.

But: $N_{th.} < N \leftarrow$ limited by available CY-orientifold geometries

- Let us focus on the most optimistic case $N_{th.} \simeq N$
- Let us also define $c_N \equiv \dfrac{\mathcal{V}^{2/3}(N)}{N} \gg 1$ as our "control parameter"

- We also need the quality of control w.r.t. curvature corrections at the "tip of the throat", summarized by $\underline{g_s M \gtrsim 4}$

[AH/Schreyer/Venken ; Junghans ; Schreyer/Venken '22]

- This leads to the most up-to-date form of the <u>"LVS Parametric Tadpole Constraint"</u>

$$N \gtrsim \mathcal{O}(1) \cdot \frac{\kappa_s^{2/3} (g_s M)^2}{\xi^{2/3} \, a_s} \ln^2 \left[\mathcal{O}(1) (a_s g_s M)^{1/4} c_N^{5/8} \Big/ \kappa_s^{1/6} \xi^{1/12} \right]$$

$$\Longrightarrow \boxed{N \gtrsim 500}$$

with most optimistic choices (e.g. $c_N = 5$)

Largest negative tadpoles in explicit geometries known at present

[Taylor/Wang '15, ..., Crino/Quevedo/Schachner/Valandro '22]

Calabi-Yau-Orientifold: $-Q = 252$ too small?

CY Orient. w. mobile D7s: $-Q = 3'332$

F-theory: $-Q \simeq 75'000$

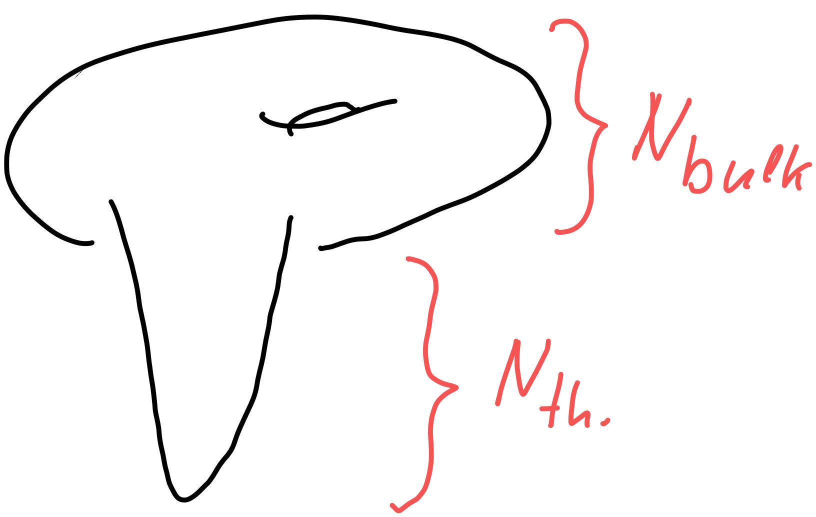
Control problems due to strongly varying g_s or to inability to ensure $g_s \ll 1$.

What is the maximal $|Q|$?

The Tadpole Problem / Conjecture

[Bena/Blaback/Graña/Lüst ; Plauschinn ; Cicoli ... Maharana ; Grimm/Heisteeg/... ; Becker/Gonzalo/Walcher/Wrase '22]

We are driven to the following situation :



$$N = N_{bulk} + N_{th.} \leq |Q|_{max}$$

$N_{th.} \gg 1$ needed for control

$\Longrightarrow$ Would like to keep N_{bulk} small !

- Will (bulk) fluxes with a small tadpole N be able to stabilize all c.s. moduli?

- The Tadpole Conjecture claims just the opposite:

> If some flux vector stabilizes a large number n of c.s. moduli, then $N_{flux} > \alpha\, n \quad (\alpha = O(1))$

Variants:
1) $\alpha > 1/3$ ("refined")
2) one may or may not require that the stabilized geometry is smooth.
3) one may or may not require $n = n_{max} = h^{2,1}$ ("strong" / "weak")

$\Rightarrow$

- in fact, 8 different conjectures
- counterexamples to the "strong" form already exist (→ Lüst/Wiesner; Coudarchet/Marchesano ...)
- precise meaning of "large n" unclear (counterexamples for $n \sim O(\text{few})$ are well-known)
- arguments in support are relatively weak:

1) K3×K3 example But: simple structure of periods
2) Example analysis/proofs at large complex structure But: Again, all rests on simple form of periods.

(well-known) argument against Tadpole Conjecture

minimum of potential: $\quad DW = \partial_z W + K_z W = 0$

$\Rightarrow$ n eqs. for n variables, with generic fcts. (periods) involved

$\Rightarrow$ expect only discrete solutions; i.e. all moduli are "generically" stabilized.

How Tadpole Conj. could still be right:

- no solutions in phys. domain of z (i.e. flux potential leads to runaway to decompactification or to singularities which are too bad to be controlled)

Interesting final point:

The Tadpole Conjecture can be made mathematically more precise by giving it a hodge-theoretic formulation

[Braun/Valandro ; Grimm et al. ; Walcher et al. '22]

Tadp. Conj. in Landau-Ginzburg model w/o Kahler moduli

$$DW = 0 \iff G = F_3 + SH_3 \in H^{2,1}(X) + H^{0,3}(X)$$

$$\text{with } F_3, H_3 \in H^3(X, \mathbb{Z})$$

$$\& \; N_{flux} = \int_X F_3 \wedge H_3$$

Def.: SUSY flux lattice: All flux choices satisfying satisfying conditions above, i.e.

$$\Lambda^{SUSY}_{(S,z)} \subset H^3(X,\mathbb{Z}) \oplus S\, H^3(X,\mathbb{Z})$$

Def.: SUSY locus: $\mathcal{M}^{SUSY}_X = \{(S,z) \in \mathcal{H} \times \mathcal{M}_X \,|\, rk(\Lambda) > 0\}$

Def.: Number of stabilized moduli: codim of $\mathcal{M}^{SUSY}_X$

Proposal: $\max\limits_{G} \left(\mathrm{codim}^{Z}_{(S,z)}(\mathcal{M}^{SUSY}_X) / |Q(G)| \right) < 1/\alpha$

Zariski dimension used $\Rightarrow$ fields stabilized by higher potential terms not counted

Summary / Conclusions

- Some of the most pressing issues in establishing a "realistic" string landscape have to do with higher-dimension operators in $d=10$.

- The possibilities for avoiding those issues depend on the availability of geometries with large negative tadpole $|Q| \gg 1$.

- Related critical issue: Can one stabilize many moduli by flux with a limited tadpole contribution?