# A local Wheeler-DeWitt Measure for the String Landscape

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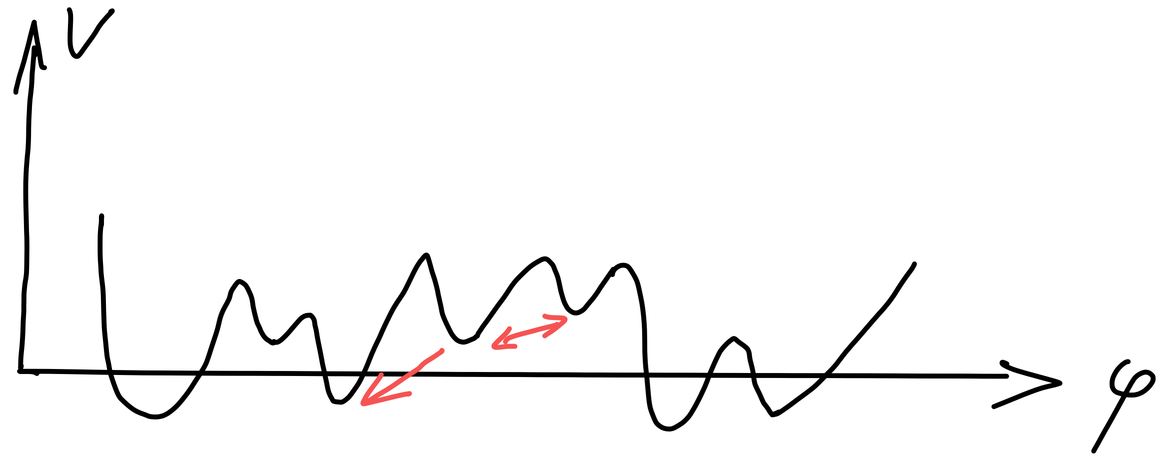
Based on work with B Friedrich, M Salmhofer, J. Strauß, J. Walcher

## Outline

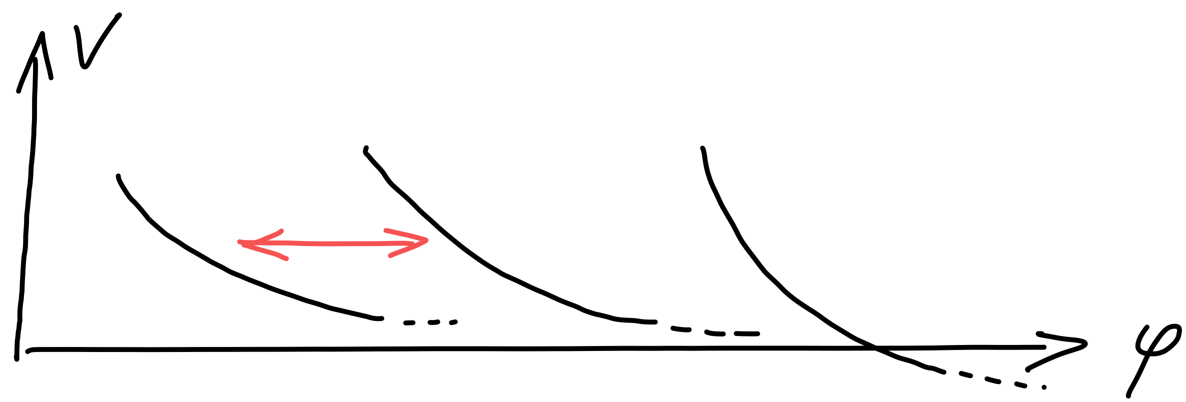
- Intro: Measure problem; Cosm. Central Dogma
- Naive QM approach; Shnirelman Theorem
- WDW approach and the role of time
- Terminal vacua; probability currents
- Towards applications ....

# The String landscape

"Classical" picture:




More skeptical "Swampland view":

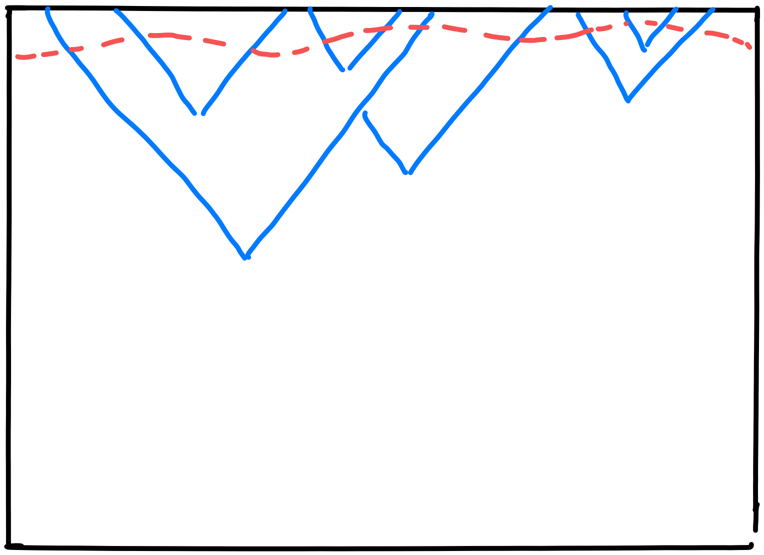



Nevertheless: Unique vacuum unlikely

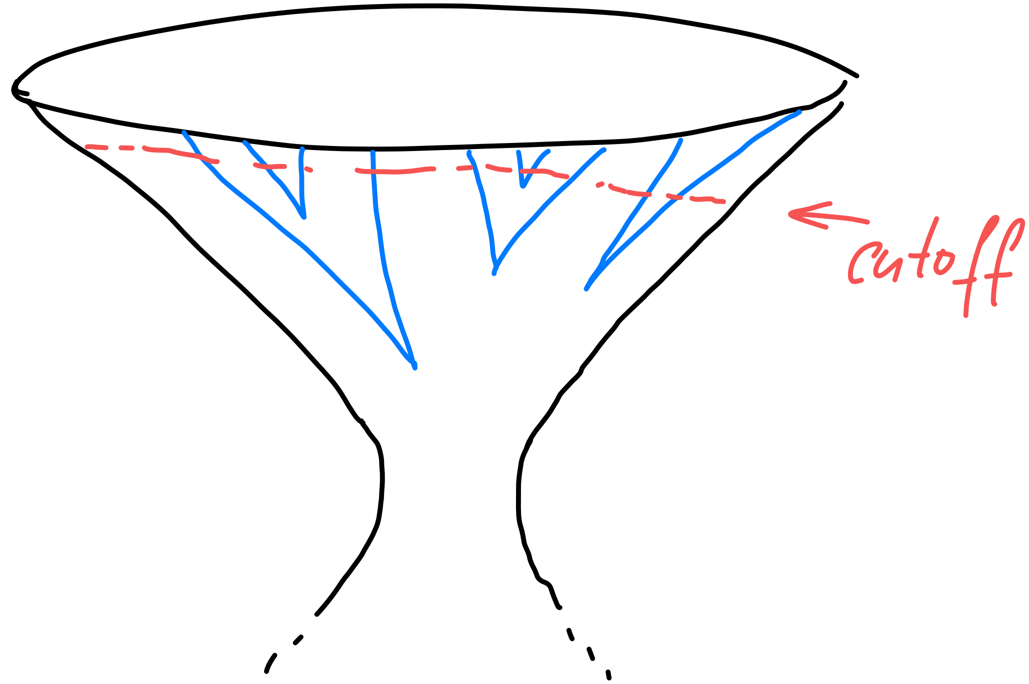
$\Rightarrow$ Need statistical predictions

# Measure Problem in Eternal Inflation

Penrose diag



Geomet. picture




- No analogous "global" picture for the skeptical view above is known
- Still, our approach will be suitable for both cases (with or without eternal inflation).

- From the beginning of the measure problem, the alternative "local view" has been discussed

Consider only what happens along worldline (or inside horizon) of a single observer

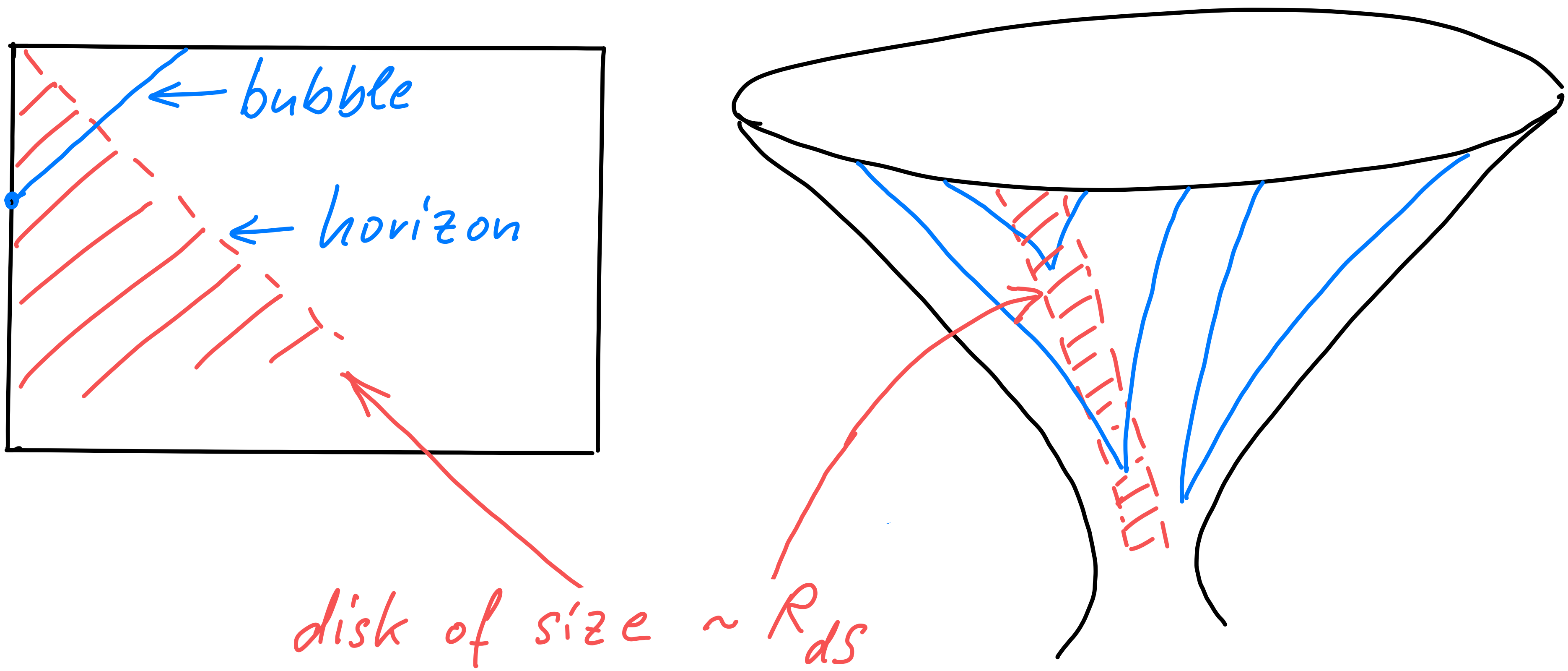
[Dyson/Kleban/Susskind '02 ; Nomura '11 ; Hartle/Hertog '11 ; Garriga/Vilenkin '12 ; ....]

- We want to formalize this appealing to the "Cosmol. Central Dogma"

dS space $\equiv$ QM system with finite number of d.o.f (set by horizon area)

[Banks '00; Susskind '21]

# "CCD - perspective"



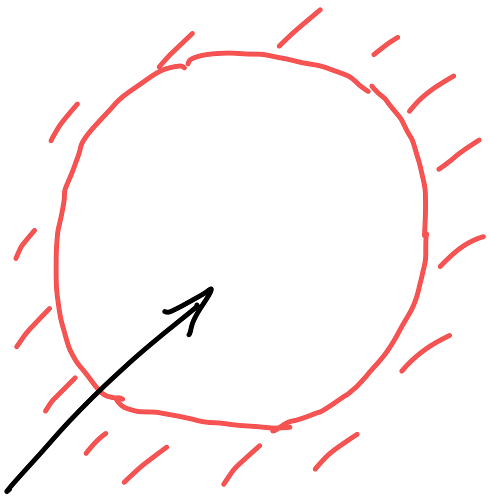

## One key motivation for this view:

Analogy with BH

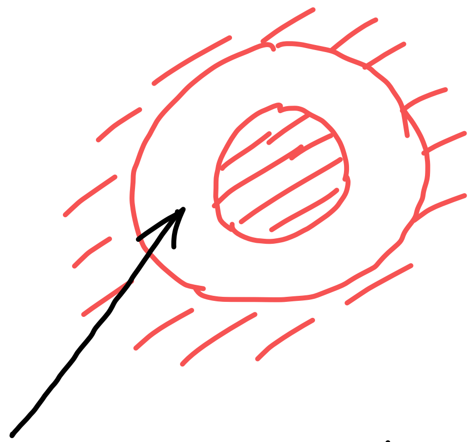
(inside corresponds to finite Hilbert space ... )

## Another key motivation:

Nariai-limit of BH in dS



empty dS space



dS space with large BH
(total horizon area and hence entropy has shrunk)

$\Rightarrow$ Excitations inside hor. $\hat{=}$ more ordered state of the whole $\boxed{\text{finite}}$ system.

# Density of states of dS

$$\frac{1}{R} \sim T \sim \frac{dE(S)}{dS} = N\frac{dE(N)}{dN} = \frac{N(E)}{\rho(E)}$$

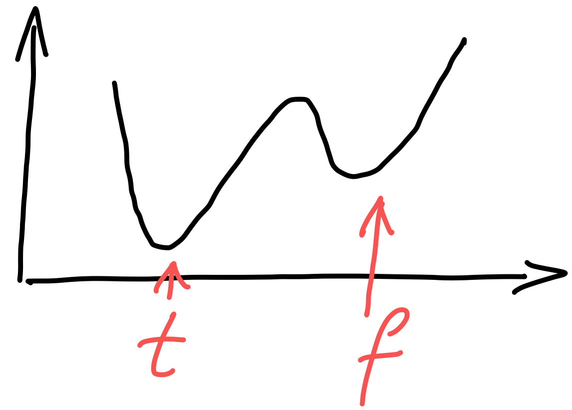
$S \sim \log(N)$ $\qquad$ $\rho \equiv dN/dE$

$\Rightarrow$ $\underline{\rho(E) \sim N}$ near maximal density

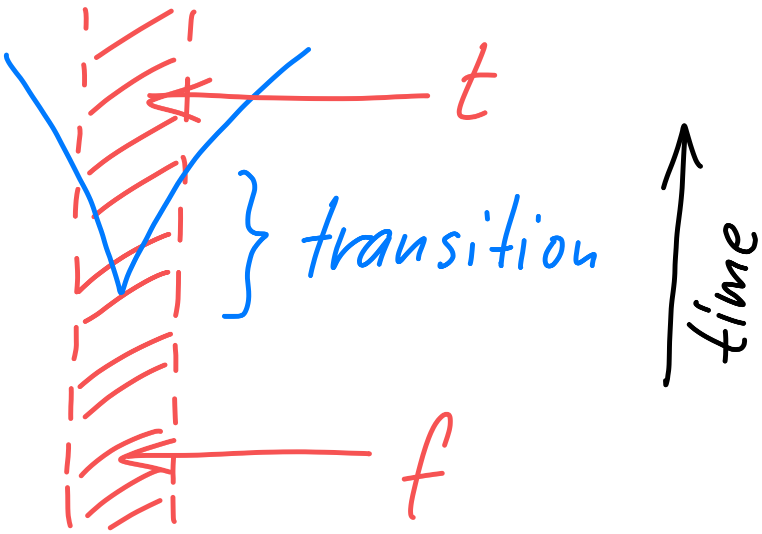
(value of $E$ irrelevant for us)


$\leftarrow \rho \sim N$

# Simplest model landscape:

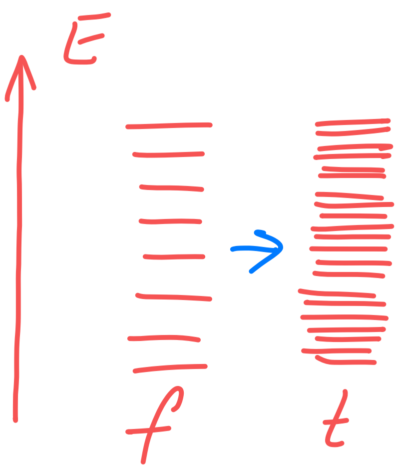



$$\mathcal{H} = \mathcal{H}_f \oplus \mathcal{H}_t$$




$$H = \begin{pmatrix} H_f & \Delta \\ \Delta^+ & H_t \end{pmatrix}$$

small entries $\sim \delta$




Bubble nucl.: $\Gamma_{sc} \sim e^{-B}$ ← "bounce"

QM model: $\Gamma_{qm} \sim \delta^2 \varrho_t$

$$\Rightarrow \quad \delta^2 \sim e^{-B}/\varrho_t$$

This $\delta$ must be large enough to prevent existence of exact eigenstates pointing mostly in $\mathcal{H}_f$ (i.e. QM-pert.-theory must break down)

$$\xRightarrow{\varrho_t \, \delta \, \gtrsim \, 1} \quad e^{-B} \gtrsim e^{-S_t}$$

(This relation always holds for both Coleman-De Luccia & Hawking-Moss transitions since $e^{B} < e^{S_f} \sim$ "recurrence time" — cf. KKLT '03.)

<u>Non-trivial consistency check!</u>

Next goal: Need to actually predict probability of observing "f" or "t"

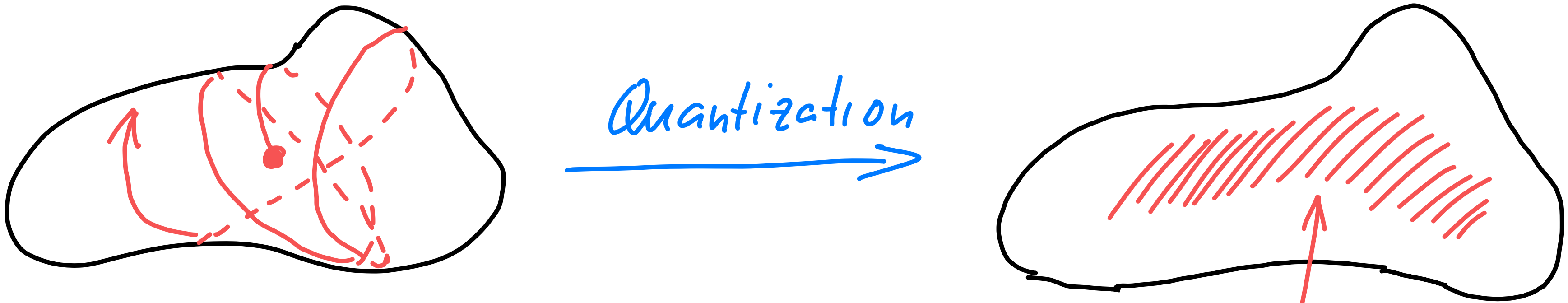
Recall: $\mathcal{H} = \mathcal{H}_f \oplus \mathcal{H}_t$ ; $H = \begin{pmatrix} H_f & \Delta \\ \Delta^+ & H_t \end{pmatrix}$

Need typical eigenvector $v$

and its projections $\|v\|_f$ ; $\|v\|_t$ on $\mathcal{H}_f$ & $\mathcal{H}_t$

Idea: Relate this to (quantum) ergodicity

Consider particle on compact manifold:

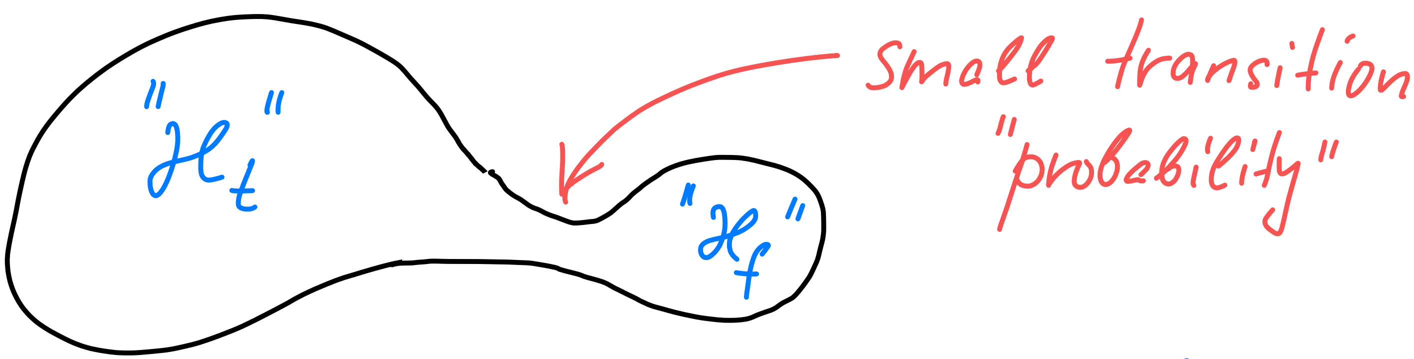



$\psi = \psi(x)$

Extension of ergodicity to QM: (<u>Shnivelman Theorem</u>)

> Eigenfcts. of Laplacian at large eigenvalue tend towards uniform distribution.

<u>"Our case:"</u>




$\Rightarrow$ expect prob. correlated with volume $\hat{=}$ dims of $\mathcal{H}_{t,f}$

⇒ Our proposed "discrete version of Shnirelman":

Typical eigenvalues of large systems $\mathcal{H}_f \oplus \mathcal{H}_t$ obey:

$$\|v\|_f \simeq \frac{N_f}{N_f + N_t}\,; \quad \|v\|_t \simeq \frac{N_t}{N_f + N_t}$$

$\dim(\mathcal{H}_{f/t})$

(Same result follows from random matrix model for eigenvector basis of $H = H_f + H_t$.)

Key observation: This purely QM claim/result is consistent with semiclass. prediction based on counting along observer worldline

Indeed, consider the ratio $R$ of observations of "f" and all observations ("f" or "t"). We just saw:

$$R_{qm} \simeq \frac{N_f}{N_f + N_t} .$$

Counting along WL in CDL tunneling setting gives

$$R_{sc} \sim \frac{\exp(-1/\Lambda_f)}{\exp(-1/\Lambda_f) + \exp(-1/\Lambda_t)} \sim \frac{N_f}{N_f + N_t} \quad \checkmark$$

<u>Next step:</u> Due to t-reparametrization invariance, we need to move from QM to WDW.

$\Longrightarrow$ $\boxed{H\psi = 0}$ as constraint.

Recall conventional WDW analysis:

Einstein-Hilbert action on $S^3 \times$ Time ;

Mini-superspace : Focus on scale factor $a = e^{\alpha}$

& scalar $\varphi(t,\vec{x}) = \varphi(t)$

$$\Rightarrow S = \int dt \left[ -e^{-3\alpha} \dot{\alpha}^2 + k e^{\alpha} + e^{3\alpha} \dot{\varphi}^2 - e^{3\alpha} V(\varphi) \right]$$

$$\Rightarrow H \Psi \simeq \left[ e^{-3\alpha} \underbrace{\left( \partial_\alpha^2 - \partial_\varphi^2 \right)} + e^{3\alpha} V(\varphi) \right] \Psi(\alpha, \varphi) \overset{!}{=} 0$$

large $\alpha$

cf. $(\partial_t^2 - \nabla^2) \Psi(x^\mu)$ of relativistic point-particle after quantization

$$\Rightarrow \boxed{\alpha \ \hat{=}\ \text{time}}$$

$\Rightarrow$ Wave fct $\Psi(\alpha, \varphi)$ tells us how $\varphi$ evolves in "time" on global dS. This has been applied to measure problem ....

[Mersini '05 ; Mersini/Perry '14]

**But:** In our opinion questionable since change of vacuum only local, not global.

[see also Podolsky/Enqvist '07 ; Hartle/Hertog '16 ;
<u>more recently:</u> Cespedes/de Alwis/Muia/Quevedo '20]

- We really need a <u>local version</u> of the WDW approach above
- Is it possible/sensible to introduce a "time" variable <u>$\alpha$</u> related to d.o.f "<u>behind horizon</u>"?

- Is classical time somehow encoded in growth of complexity in "horizon-Hilbert-space"?

  $\rightarrow$ cf. detailed discussion in paper....

  upshot: Too many open issues – dismiss for now!

- Instead: Use "purist's" WDW perspective

  Time emerges only through correlations of "light" quantum variable and "heavy" semi-classical variable. [e.g Banks '85]

  $\Downarrow$

  Apply this to each dS patch & observer independently

$\Longrightarrow$ No need for global "time" variable $\alpha$ or $a = e^{\alpha}$

## Some details (review of "Banks '85")

- Single-particle Hamiltonian: $H_0 = \frac{p^2}{2M} + V(x)$

- WKB: $H_0 \psi_E = E \psi_E$

$$\psi_E(x) \sim \left[2M(E-V(x))\right]^{-1/4} \exp\left\{ \pm i \int^x \sqrt{2M(E-V(x'))}\, dx' \right\}$$

- Add light d.o.f and impose WDW constraint:

$$H\psi = 0 \;, \qquad H = H_0(x) + H_\ell(x,y)$$

Ansatz $\quad \psi = \sum_E \psi_E(x)\,\chi_E(x,y)$

$$\Rightarrow 0 = \sum_E \Big[ E\psi_E\chi_E - \underbrace{\frac{1}{2M}\big(2\partial_x\psi_E\,\partial_x\chi_E + \dots\big)}_{\partial_t \chi_E} + \psi_E H_\ell \chi_E \Big]$$

with $\dfrac{dt}{dx} \sim \dfrac{1}{\sqrt{2M(E-V(x))}}$

$\Rightarrow$ Schrödinger eq. for $\chi_E(x,y)$.

$x \;\hat{=}\; t$

Our (modest) contribution: Demonstration, that logic survives the addition of many weakly-coupled (hidden) d.o.f., eg from horizon: $\quad H = H_0 + H_\ell + \underline{\underline{H_h}}$

$\Rightarrow$ Schrödinger eq. for $\mathcal{H}_e$ emerges as before

$\Rightarrow$ Resulting picture

$$\mathcal{H} = \bigoplus_i \mathcal{H}_i^{ds} \quad ; \quad H = \sum_i H_i \quad ; \quad H\psi = 0$$

$$\|\psi\|_i^2 \sim \frac{N_i}{\sum_j N_j}$$

(consistent with "Shnirelman" & counting along observer WL)

No time variable needed!

(time emerges only locally, if an observer uses a semiclassical clock)

Next step: <u>Add "terminal vacua"</u> (AdS and Mink)

- since $N_{i,\,ter} = \infty$, we expect $\|\psi\|_i / \|\psi\|_{j,\,ter} = 0$, but $\|\psi\|_i / \|\psi\|_j$ could stay finite
- <u>But a different problem emerges:</u>

$\psi$ stationary $\Rightarrow$ transitions $i \to j,\,ter$ and $j,\,ter \to i$ must be present.

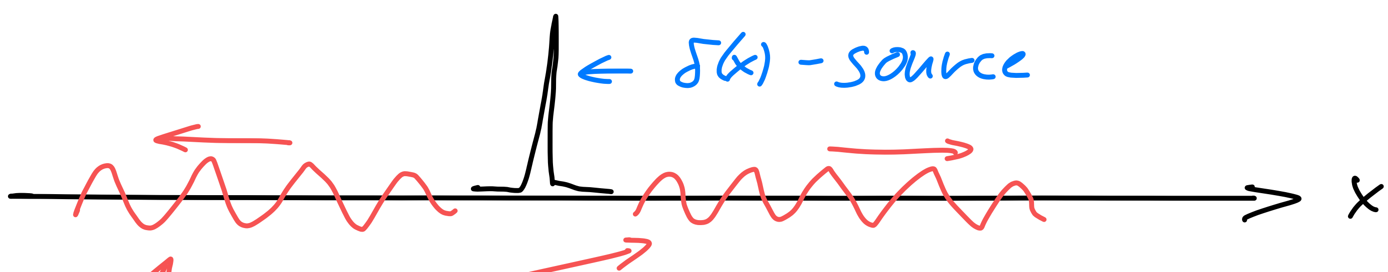
But the latter are <u>semiclassically forbidden</u>!

(see however Nomura '12 ...)

We propose the following way out:

$$\underbrace{\text{Hartle / Hawking, Linde / Vilenkin creation of } S^3\text{'s} \Longrightarrow \text{dS vacua} \;\underset{\cancel{\Longleftarrow}}{\Longrightarrow}\; \text{Terminals}}_{\text{stationary flow}}$$

<u>Toy model (for illustration):</u> 1-particle QM




currents $\sim i(\nabla\psi^*)\psi + h.c.$ ; flux goes to infinity (like in AdS/Mink)

## Explicit solutions of corresp time-indep Schrödinger eq:

$$(\partial_x^2 + E)\psi(x) = c\,\delta(x) \Rightarrow \psi \sim \begin{cases} e^{ikx} & (x>0) \\ e^{-ikx} & (x<0) \end{cases}$$

$\Rightarrow$ currents $j \begin{matrix} >0 \\ <0 \end{matrix}$ $\quad [k \equiv \sqrt{E}]$

[opposite flux also possible $\Rightarrow$ boundary cond.s decide whether $\delta(x)$ is a source or sink]

We need a "discrete version of the above probability current"

(This is well-known, e.g. de Andrada '92.)

## "discrete" current:

$$\underbrace{J_{mn}} = i\left(H_{nm}\,\rho_{mn} - \rho_{nm}\,H_{mn}\right) \quad \text{where} \quad H_{nm} = \langle n|H|m\rangle$$

$$\rho_{nm} = \langle n|\rho|m\rangle$$

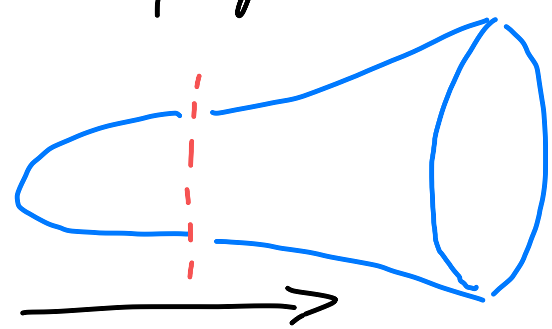
quantifies prob flux $|n\rangle \to |m\rangle$

(In time-dep. case easy to see: $\partial_t |\langle n|\psi\rangle|^2 = \sum_m J_{nm}$)

$\Rightarrow$ Our proposal for most fund WDW eq:

$$\boxed{H\psi = \chi}$$

encodes creation of spheres from noting:



"time"

## More explicitly:

Let $\{|n\rangle\}$ be basis of $\mathcal{H}_i^{dS}$

-"- $\{|a\rangle\}$ -"- $\mathcal{H}_x$ ← dS or terminal

$$J_{i\to x} = \sum_{a,n} J_{an} = 2\,\mathrm{Im}\Big(\sum_{a,n} \langle a|H|n\rangle\langle n|\underbrace{\psi\rangle\langle\psi}_{\rho}|a\rangle\Big)$$

$$= 2\,\mathrm{Im}\big( {}_x\langle\psi|H|\psi\rangle_i \big)$$

$|\psi\rangle$ projected on $\mathcal{H}_x$ and $\mathcal{H}_i$

## At the same time:

$$J_{i\to x} = p_i \Gamma_{i\to x} - p_x \Gamma_{x\to i}$$

decay rates (from above or from CDL)

Strength of source at $\mathcal{H}_i^{dS}$.

$$J_i = \sum_x J_{i\to x} \sim \begin{cases} \exp(S_i) & \text{"HH"} \\ \exp(-S_i) & \text{"LV"} \end{cases}$$

Thus, finally:

$$\boxed{J_i = \sum_{j\in dS} \left(p_i \Gamma_{i\to j} - p_j \Gamma_{j\to i}\right) + p_i \sum_{y\in Ter} \Gamma_{i\to y}}$$

to be solved for $\{p_i\}$

$\Rightarrow$ Stationary flow through the landscape of vacua.

$\Rightarrow$ Sources are truly necessary since terminals exist.

- Not surprisingly, technically similar proposals exist in the literature

  [Garriga, Schwarz-Perlov, Vilenkin, Winitzki '05 .. '12]

- One of the closest: Count vacua along observer WL; Introduce $J_i$ as initial distribution of trajectories or as distribution of "returns" from terminals

Our interpretation is fundamentally different:

The probabilities $p_i$ characterize the wave fct of the universe $\psi$, which solves the fundamental inhomogeneous WDW eq $H\psi = \chi$

**Our claim:** This is not just one of the many ways of counting, but it is the unique measure emerging from CCD and WDW

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**A simple toy model:** dS vacua "1, 2" with $S_1 < S_2$

+ one terminal "T"

$$\Rightarrow \begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \underbrace{\begin{pmatrix} \Gamma_{1\to 2} + \Gamma_{1\to T} & -\Gamma_{2\to 1} \\ -\Gamma_{1\to 2} & \Gamma_{2\to 1} + \Gamma_{2\to T} \end{pmatrix}}_{\text{to be inverted}} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

$\Rightarrow \{p_i\}$

$\Longrightarrow$ Ratios of $p_i$ are a prediction

(recall: $S_1 < S_2$)

LV:

$$\frac{p_1}{p_2} \simeq \frac{\Gamma_{2\to T}\, e^{-S_1}}{\Gamma_{1\to 2}\, e^{-S_1} + \Gamma_{1\to T}\, e^{-S_2}}$$

HH:

$$\frac{p_1}{p_2} \simeq \frac{\Gamma_{2\to T}\, e^{S_1}}{\Gamma_{1\to T}\, e^{S_2}}$$

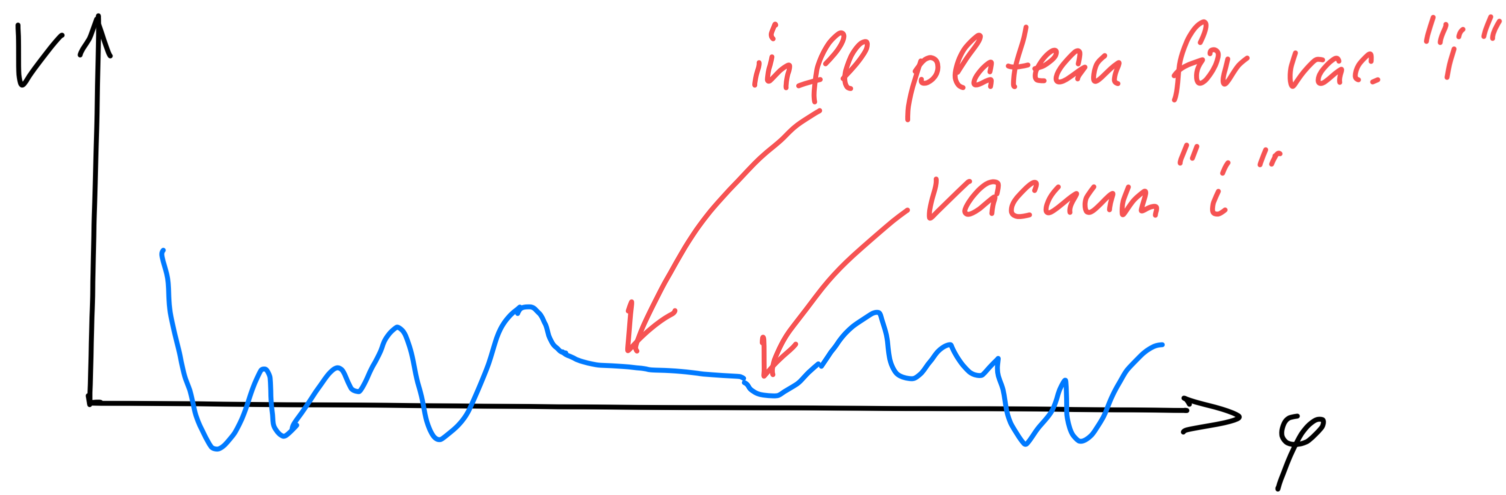
LV: The explicit result $(p_1/p_2 \gtrsim 1)$ depends on which of two large ratios $\Gamma_{1\to T}/\Gamma_{2\to T}$ & $e^{S_2}/e^{S_1}$ wins.

Finally, we want to ask about

**Probabilities for "anthropic" observers**

(in post-inflationary cosmology)

$\Rightarrow$ Need probability $O_i$ that vacuum "i" is not just populated but populated through tunneling to or creation of its inflationary plateau:



$$\Rightarrow \quad O_i \sim w_i \left( f_i \Gamma_i + \sum_j p_j f_{ji} \Gamma_{j\to i} \right)$$

prob for observers in "i" after reheating

fractions of creation/tunneling processes leading to "i" and going through inflationary plateau.

.... it would be interesting to attempt doing some preliminary pheno studies based on this....

# Summary / Conclusions

- Reasonable possibility: Quantum-mechanically, there is no infinity of d.o.f. behind the dS horizon ("Cosm. Central Dogma")
- Together with WDW, this implies a "stationary probability flow" through the landscape
- Sources (LV or HH) are necessarily part of this picture

  ⇒ fundamentally unique "Local WDW Measure"